De
Orbitis Cometarum
ex obseruatis eorum tribus locis geocentricis analytice determinandis

Articulus 1

Inuenta virium centralium Theoria, quae incrementa coeperint Studia ~~Mathem~~ Physico-Mathematica, adeo haec iam vulgo nota sunt, vt superfluum existimem rem toties a summis viris tractatam, totque exemplis comprobatam in medium adducere. Erant quidem qui totam hanc virium legem in dubium vocarunt, sed postquam Io. Bernoullius in litteris ad Hermannum datis quod Newtonus supposuerat ex principiis Calculi Integralis solidissime ostendit, Traiectoriam, quam corpus a viribus centralibus sollicitatum describit esse necessario Sectionem conicam, si eae vires fuerint quadratis distantiarum a puncto quodam in plano Traiectoriae existente inuerse proportionales (1), legis Newtonianae existentiam paene omnes agnouerunt. Interea licet dictarum virium existentia manifesta sit, attamen ~~eorundem~~ ~~aduocabu-~~ larundem applicatio ad calculandas corporum coelestium orbitas tantis saepe difficultatibus laborat, vt consultius

(1) Extrait de la reponse de M. Bernoulli à M. Hermann datée de Basle le 7. Octobre 1710. Io. Bernoulli Opera Tom. I p. 470

Magyar Nemzeti Levéltár Országos Levéltára (MNL OL)
Magyar Királyi Helytartótanács, Departamentum litterario-politicum (C 67), 1796. F. 9, p. 218.

plerique putent ad methodos indirectas et graphicas delineationes confugere, quam praestricto universali motus principio, et via analytica procedendo, tot calculorum labores exantlare.

Praecipue se difficultas ista in determinatione orbitarum a Cometis descriptarum manifestat, in cujus quidem Problematis solutione, praeterquam quod ad aequationes altissimorum graduum perveniatur, illud etiam incommodi occurrit, et in quo Cometae ab ordinariis Planetis cumprimis differunt, quod cum tempora periodica corporum in Ellipsi incedentium a longitudine axis maioris dependeant, Cometae autem tunc tantum, dum circa perihelia, seu in vicinia solis versantur a nobis conspici possint, nisi horum tempora periodica in antecessum cognoscantur, quemadmodum nihil de axe transverso, ita neque de orbitis eorum ellipticis quidquam concludi possit.

Sed commode fit, ut hae ipsae cometarum orbitae sint Ellipses maxime eccentricae, ita ut vix assignari possit error qui committitur, si earum eccentricitas semiaxi maiori aequalis statuatur; Cum igitur in isto casu Ellipses quam proxime ad Parabolam accedant, exigui illi orbitarum arcus quos cometae circa solem describunt quando oculis nostris obiiciuntur, tuto iam pro parabolicis haberi possunt, id quod et calculum concinniorem reddit, et sufficientes conditiones suggerit ad habenda omnia, si demas periodicum tempus, earundem orbitarum elementa: cum, uti notum est, ad describendam parabolam praeter parametrum et duo puncta nihil amplius, ad Ellipsim vero etiam axis maior requiratur.

Verum quidem est, ut suo loco patebit, etiam in hypothesi motus parabolici perveniri ad aequationes, quarum radices perfecte exhiberi nequeunt, at dummodo pateat methodus ad easdem pro lubitu accedendi, poterunt eae tam exacte inveniri, ut error ex duplici causa in elementa orbitae redundans ad unicum tantum, eumque vix sensibilem, hypothesi nempe motus parabolici debitum, reducatur. Atque hoc est proprie, quod inde a Newtoni temporibus omnium Mathematicorum, qui vires suas in determinandis orbitis cometarum experiri voluerunt industriam exercuit, et plerumque elusit, licet quidem negari nequeat, multum luminis ab eorum conatibus in totam hanc materiam processisse, ipsumque summum virum La Grangium, cuius hic cogitationes persecuti sumus non tam novam methodum invenisse, quam veterem, et praecipue illam quam novissime Condorcetus exposuit (2) perfectiorem reddidisse (3). Sed his breviter tantum dictis ad ipsam Theoriam Cometarum progrediemur, prius tamen pauca quaedam, quorum postea erit nobis usus exponemus.

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(2) Essai sur la Théorie des Comètes.

(3) Memoire sur le Probleme de la détermination des orbites des Comètes d'après trois observations.

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Articulus II

1. Si corporis S (Fig. 1) in distantia quadam pro unitate assumta vis attractrix fuerit $= F$, tum circa hoc moueatur aliud corpus C in Traiectoria DC viribus centralibus, quae sint inverse vt quadrata distantiarum eiusdem a corpore S, erit vis qua proiectile C in distantia $SC = r$ urgetur versus $S = \frac{F}{r^2}$; posita enim ea vi $= p$ exit $p : F = 1 : r^2$, adeoque $p = \frac{F}{r^2}$.

2. Si nunc situs proiectilis C referatur ad tres coordinatas $SP = x$, $PM = y$, $MC = z$, quarum postrema MC sit ad planum reliquarum duarum, MP autem ad PS rectangula, seu quod idem est, si proiectile C referatur ad tres coordinatas SP, PM, MC tribus axibus SX, SY et SZ sibi inuicem in eodem puncto S perpendiculariter insistentibus parallelas, denotante CS vim attractricem corporis S in distantia et directione $CS = r$, rectae SP, PM, MC designabunt effectus vis attractricis CS iuxta directiones axium rectangulorum SX, SY, SZ, ita vt si vires hae vocentur P, Q, R, habeatur

$$P = \frac{Fx}{r^3}$$

$$Q = \frac{Fy}{r^3}$$

$$R = \frac{Fz}{r^3}$$

3. Hinc cum in genere celeritas elementaris vi acceleratrici p debita exprimatur aequatione $dc + pdt = 0$, sitque $c = \frac{ds}{dt}$, erit quoque

$$d.\frac{dx}{dt} + \frac{Fx}{r^3}dt = 0$$

$$d.\frac{dy}{dt} + \frac{Fy}{r^3}dt = 0$$

$$d.\frac{dz}{dt} + \frac{Fz}{r^3}dt = 0$$

$$d.\frac{dr}{dt} + \frac{F}{r^2}dt = 0$$

seu posito dt pro constante

$$\frac{d^2x}{dt} + \frac{Fx}{r^3}dt = 0$$

$$\frac{d^2y}{dt} + \frac{Fy}{r^3}dt = 0$$

$$\frac{d^2z}{dt} + \frac{Fz}{r^3}dt = 0$$

$$\frac{d^2r}{dt} + \frac{F}{r^2}dt = 0$$

quae sunt aequationes pro celeritatibus elementaribus vi attractrici p iuxta directiones x, y, z et r in distantiis x, y, z et r debitis.

4. Porro si vis tangentialis, qua corpus in quouis Traiectoriae suae puncto urgetur dicatur T, erit aequatio eiusdem celeritatem tangentialem exprimens ~~haec~~ sequens $d.\frac{ds}{dt} + Tdt = 0$, seu $d.\frac{\sqrt{(dx^2 + dy^2 + dz^2)}}{dt} + Tdt = 0$. At vero vis tangentialis non est functio distantiae corporis a puncto attrahente, sed penitus arbitraria, nisi igitur ea a posteriori cognoscatur, nihil etiam poterit de celeritate corporis in axem incidentis cognosci, ex quo simul legum Keplerianarum necessitas patet. His legibus suppositis sequentia reperiuntur.

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5. Sint R et r duorum circulorum radii et ratio diametri ad peripheriam exprimatur per $1:\pi$, Area circuli cuius radius est R sit A, alterius A', erit $A = \pi R^2$, $A' = \pi r^2$. Hinc si supponatur dictos circulos esse orbitas planetarias, tempora vero quibus describuntur T et T' erit per legem Keplerianam $T:T' = \frac{A}{\sqrt{R}} : \frac{A'}{\sqrt{r}}$ seu $T:T' = \frac{\pi R^2}{\sqrt{R}} : \frac{\pi r^2}{\sqrt{r}} = \pi R^{\frac{3}{2}} : r^{\frac{2}{2}}$ adeoque $T = \pi R^{\frac{3}{2}} . \frac{T'}{\pi r^{\frac{3}{2}}}$. Si iam pro quodam radio r cognitum sit tempus periodicum T', erit quantitas $\frac{T'}{\pi r^3}$, determinata, ea igitur posita $= M$ erit $T = \frac{\pi R^{\frac{3}{2}}}{M}$, expressio temporis quo describitur circulus cuius radius est R. Est vero celeritas corporis in circulo incedentis constans, quare cum peripheria circuli posito radio $= R$ sit $2\pi R$ erit celeritas proiectilis in circulo cuius radius est R incedentis $c = \frac{2\pi R}{\frac{\pi}{M} R^{\frac{3}{2}}} = \frac{2M}{\sqrt{R}}$.

6. Facile quoque inveniuntur tam periodica tempora, quam celeritates corporis in Elipsi incedentis; nam positis duarum elipsium semiaxibus coniugatis a, b, a', b', parametris p et p' areis A et A', et temporibus periodicis t, t' erit per legem Keplerianam $t:t' = \frac{A}{\sqrt{\frac{1}{2}p}} : \frac{A'}{\sqrt{\frac{1}{2}p'}}$, seu ob $p = \frac{2b^2}{a}$, $p' = \frac{2b'^2}{a'}$ et posita ratione diametri ad peripheriam $= 1:\pi$, $A = \pi ab$, $A' = \pi a'b'$. $t:t' = \frac{\pi ab}{\sqrt{\frac{b^2}{a}}} : \frac{\pi a'b'}{\sqrt{\frac{b'^2}{a'}}} = \pi a^{\frac{3}{2}} : \pi a'^{\frac{3}{2}}$, unde $t = \pi a^{\frac{3}{2}} . \frac{t'}{\pi a'^{\frac{3}{2}}}$ et posito $\frac{\pi a'^{\frac{3}{2}}}{t} = m$

$t = \frac{\pi a^{\frac{3}{2}}}{m}$, aut etiam $t = n\pi a^{\frac{3}{2}}$ si fiat $\frac{1}{m} = n$

Ex quo patet illud quod in principio diximus, tempora periodica planetarum et cometarum tantum ab axe maiore, non item etiam a foco dependere, esseque proinde tempora periodica in Elipsibus quomodocunque excentricitate differentibus aequalia, si earum axes maiores fuerint aequales. Cum igitur circulus sit ellipsis cuius eccentricitas $= 0$, erunt tempora periodica in circulo et elipsi aequalia, si diameter prioris fuerit aequalis axi maiori posterioris, adeoque posse haberi $M = m = \frac{1}{n}$

7. Ad inveniendum valorem m assumatur orbita telluris, eiusque tempus ~~cognitum~~ periodicum exprimatur in diebus naturalibus temporis medii, et fractionibus diei decimalibus, erit $T = 365,25659$; posito deinde semiaxe maiore orbitae terrae, seu eiusdem distantia media a Sole $= 1$, ob $t = n\pi a^{\frac{3}{2}}$ erit $n = \frac{t}{\pi} = \frac{365,25659}{3,14159265} =$ $116,2648$

8. Quod si iam reperienda sit celeritas proiectilis in ellipsi incedentis, sit (fig 2) FC arcus ellipticus, CD, arcus circuli radio vectore $SC = r$ descripti, SN radius vector priori infinite propinquus, adeoque CN et CD elementa arcus elliptici et circularis; tempus quo percurritur $CN = dT$, quo $CD = dt$, axis ellipsis maior $= 2a$, distantia foci a vertice $= f$, parameter $= p$ erit $\frac{1}{2}p = \frac{2(af - f^2)}{a}$ adeoque

$$dT = \frac{n.CSN\sqrt{a}}{\sqrt{(2af - 2f^2)}}$$

$$dt = \frac{n.SCD}{\sqrt{r}}$$

quare positis per CN et CD celeritatibus K et c ob $K = \frac{CN}{dT}$, $c = \frac{CD}{dt}$ habebitur

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$$k = \frac{CN.\sqrt{(2af - 2f^2)}}{n.CSN\sqrt{a}} \quad (n. 6)$$

$$c = \frac{CD.\sqrt{x}}{n.CSD} \quad (n. 5)$$

adeoque $k : c = CN\sqrt{(2af - f^2)} : CD\sqrt{ax}$, ob sectores SCN et SCD aequales. Est vero ut Lambertus ostendit $CD = CN.\sin CND = \frac{\sqrt{(af - f^2)}}{\sqrt{(ax - x^2)}}$, (4). quare $k : c = \sqrt{2(ax - x^2)} : \sqrt{ax}$, sed $c = \frac{2}{n\sqrt{x}}$, (n. 5). quare

$$k = \frac{2\sqrt{2(ax - x^2)}}{x\sqrt{a}}.$$

9. Hinc sequentia eximia corollaria deducuntur, 1°. quod si duorum aut plurium corporum proiectorum tempora periodica fuerint aequalia, eos in iisdem a sole distantiis easdem celeritates habere. Cum enim $k = \frac{2\sqrt{2(ax - x^2)}}{x\sqrt{a}}$, corpora autem quorum tempora periodica sunt aequalia habeant axes maiores orbitarum suarum aequales (n. 6), expressio k identica exit, quotiescunque in duobus aut pluribus casibus praeter semiaxes a, fuerint etiam distantiae ~~aequ~~ a sole x aequales. 2do celeritatem proiectilis in circulo aequalem esse ei celeritati, quam haberet in vertice axis minoris Ellipseos, cuius Axis maior esset diametro circuli aequalis; nam celeritas in vertice axis minoris Ellipseos ob $x = \frac{1}{2}a$ est $k = \frac{2\sqrt{2}}{n\sqrt{a}}$ n. praec. at celeritas in circulo cuius radius ponitur esse $= \frac{a}{2}$ est itidem $\frac{2\sqrt{2}}{n\sqrt{a}}$, n. 5. patet igitur propositum.

(4) Insigniores orbitae Cometarum proprietates § 173.

10. Quo etiam celeritas corporis in parabola incedentis inveniatur, repraesentet ut antea TC arcum parabolicum, CD circularem, sitque tempus quo describitur elementum arcus parabolici $CN = dT$, et celeritas $= k$, tempus autem quo describitur elementum arcus circularis $CD = dt$, celeritas $= c$, erit $dT = \frac{n.CSN}{\sqrt{\frac{1}{2}p}}$, $dt = \frac{n.CSD}{\sqrt{SC}}$; est vero $k = \frac{CN}{dT}$, $c = \frac{CD}{dt}$, quamobrem

$$k = \frac{CN.\sqrt{\frac{1}{2}p}}{n.CSN}$$

$$c = \frac{CD.\sqrt{CS}}{n.CSD}.$$

Sed posito in A vertice parabolae, habetur ex cognitis huius proprietatibus $AS = CS.(\sin CND)^2 = \frac{CS.CD^2}{CN^2}$, unde $CN.\sqrt{AS} = CD.\sqrt{CS}$, ~~sed~~ seu ob $AS = \frac{1}{4}p$, $CN.\sqrt{\frac{1}{4}p} = CD.\sqrt{CS}$, et $CN.\sqrt{\frac{1}{2}p} = \frac{CD.\sqrt{CS}}{\sqrt{\frac{1}{2}}}$, hos igitur valores in expressionibus celeritatum k et c substituendo, habebitur ob sectores CSN et ~~[illegible]~~ CSD aequales $k : c = \sqrt{2} : 1$, quare cum praeterea sit $c = \frac{2}{n\sqrt{CS}}$ erit posito $CS = x$,

$$k = \frac{2\sqrt{2}}{n\sqrt{x}}$$

Quae celeritas, uti et illa, quam n. 8 invenimus pro ellipsi, ob introductam quantitatem n exprimit spatium uno die naturali motu aequabili confectum, seu quod uno die naturali conficeretur ~~[illegible]~~, si in distantia x corpus ~~[illegible] cessaret~~ versus centrum virium urgeri cessaret.

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Articulus III.

11. In hac sectione proposuimus nobis exponere methodum, qua liceat invenire directe elementa orbitae cometarum. Haec vero sequentibus capitibus absolvuntur. 1° Loco nodi ex sole visi. 2do Inclinatione orbitae. 3° Loco Perihelii, 4to distantia eiusdem ~~per~~ in perihelio. 5° tempore appulsus ad perihelium 6° denique motus eiusdem directione.

Hunc in finem sit in (fig. 3) S centrum Solis, WTH Ecclipticae, T locus terrae in Ecclipticae, AQ axis aequinoctiorum, C locus cometae indeterminatus pro tempore indeterminato t, NC eius orbita, KΩ communis plani orbitae cum plano Eclipticae intersectio, CE perpendicularis ad communem intersectionem KΩ in plano orbitae ducta, CM perpendicularis ad planum Ecclipticae, adeoque EM perpendicularis ad KΩ in plano Ecclipticae et CEM = i angulus inclinationis orbitae. Iam sumta linea aequinoctiorum AQ pro axe abscissarum, et ducta MP ad MS perpendiculari, erunt SP, PM, MC tres coordinatae rectangulae locum cometae determinantes, quas designabimus per x, y, z. Ducta porro ~~per~~ perpendiculari TV ad eandem ~~rectam~~ axem AQ, rectae SV et VT erunt binae coordinatae rectangulae locum telluris determinantes, quas designabimus per X et Y. His positis, et facto radio vectore cometae SC = r, radio vectore Eccliphicae ST = R, habebuntur sequentes duae aequationes

$$r = \sqrt{(x^2+y^2+z^2)}$$
$$R = \sqrt{(X^2+Y^2)}$$

Sit deinde Tℒ recta per centrum terrae transiens et ad axem aequinoctiorum AQ parallela; Tp, pM, ME tres coordinatae rectangulae ad locum cometae ex centro terrae visi, sitque Tp = ξ, pM = η, CM = ζ, radius vector geocentricus TC = ϱ habebitur

$$\varrho = \sqrt{(\xi^2+\eta^2+\zeta^2)}$$

Quo omnis ambiguitas signorum evitetur, abscissas x quae versus aequinoctium vernum A, ordinatas y quae versus orientem, et denique ordinatas z quae versus septentrionem diriguntur semper pro positivis habebimus; quae in contrarias plagas pro negativis, hac cautela cum La Grangio adhibita diversitas signorum nullam in calculo molestiam facesset.

12. Cum iam locus verus cometae per tres coordinatas rectangulas x, y, z determinetur, in eo erit elaborandum ut hae coordinatae exprimantur per quantitates notas; at vero ~~[illegible]~~ nihil aliud potest cognosci quam 1° Longitudo terrae ex tabulis Solis pro dato momento observationis cometae. 2do Longitudo et latitudo geocentrica cometae, 3° Distantia Solis a tellure seu radius vector Ecclipticae pro eodem momento observationis. Videamus igitur quo pacto ex his ad solutionem problematis trium coordinatarum x, y et z perveniri possit.

Longitudo terrae pro dato momento observationis ~~si~~ seu angulus TSV sit - - - - - - - - = A
= α
Longitudo cometae geocentrica MTp - - - - - = β
Latitudo eius geocentrica MTC - - - - - = λ
Radius vector terrae tempore observationis - - -

Hoc posito cum sit ~~ST = R cos A~~. SV = ST cos VST, VT = ST . sin VST, erit facta substitutione:

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$X = R\cos A$

$Y = R\sin A$.

eademque ratione ob $CM = TC.\sin\beta$, $TM = TC\cos\beta$, $MP = TM.\sin\alpha$, $Tp = TM\cos\alpha$, facta substitutione obtinebitur

$\xi = \varrho\cos\alpha.\cos\beta$

$\eta = \varrho\sin\alpha.\cos\beta$

$\zeta = \varrho\sin\beta$

Est autem, ut ex inspectione figurae manifestum fit $x = X+\xi$, $y = Y+\eta$, $z = \zeta$, itaque substituendo valores quantitatum X, Y et ξ, η, ζ

(A)... $x = R\cos A + \varrho\cos\alpha.\cos\beta$

$y = R\sin A + \varrho\sin\alpha.\cos\beta$

$z = \varrho\sin\beta$.

Harum aequationum ope si pro quodam observationis momento cognita esset distantia Cometae geocentrica R, haberetur locus cometae verus omnimode determinatus, cumque ut postea patebit hinc reliqua eius orbitae elementa dependeant, nihil ad perfectam Theoriam motus parabolici amplius deesset. Cel. Lambert cui plurimum debet cometarum Theoria, insignem de Heliocentricis geocentricisque Cometarum distantiis propositionem demonstravit, quod nempe distantia cometae a sole eadem sit cum distantia telluris a sole, quando motus illius apparens idem est cum vero, quodque eadem distantia cometae a sole proxime accedat ad distantiam telluris a sole, si motus cometae apparens proxime accesserit ad verum.(s) Hoc Theorema postea etiam Hennertus demonstravit, atque

(s) Nouveaux mémoires de l'academie Royale de Berlin Tome II.

ad detegendas motus cometarum proprietates traduxit. Verum duo sunt praecipue, quae efficiunt, quo minus Theorematis alias excellentis magnus usus esse possit: primum quod distantiae cometarum periheliae plerumque multum differant a distantia solis a tellure, quales sunt cometarum qui ad annos 1231, 1585, 1652, 1684, 1707, 1718, 1729, 1743, 1747, 1748, ~~1~~ 1760, 1762 referuntur; deinde quod expressio distantiae ~~in~~ geocentricae indeterminatae cometarum involvat angulum, qui per observationem non datur, ita ut ne quidem debitae approximationi locus relinquatur. Sequenti methodo his incommodis occurretur.

13. Lemma.

Planorum CNS et WIH fig (3) per commune quoddam punctum ~~[illegible]~~ S transeuntium communis intersectio est KS, recta QS sub angulo $QSK = h$ ad communem intersectionem KS inclinata est axis, S origo abscissarum, denique inclinatio planorum est $CEM = i$; quaeritur aequatio ad planum CNS.

Solutio

Ducta CM ad planum WIH, et MP ad QS perpendiculari erunt tres coordinatae rectangulae $SP = x$, $PM = y$ et $MC = z$; hoc posito habebitur $CM = ME\,\mathrm{Tang}\,i$; ducatur PR parallela ad ME, et producta ME fiat $EF = PR$, erit PREF parallelogrammum rectangulum ob angulum $FEQ = 90°$. Hinc $ME = MF - EF = MF - PR$; sed $MF = y\cos h$, et $PR = x\sin h$, quam CM seu $z = (y\cos h - x\sin h)\tan i$; seu

$z = y\cos h\tan i - x.\sin h.\tan i$.

Sit brevitatis gratia $\cos h\tan i = B$, $\sin h.\tan i = C$, ~~[illegible]~~ habebitur

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$$z = By - Cx$$

aequatio ad planum quaesita.

14. Posito nunc quod planum Lemmatis sit planum orbitae cometae ad planum Eclipticae relatum, sitque QQ axis aequinoctiorum, erit h longitudo nodi, et substitutis in aequatione $z = By - Cx$ loco x, y, z valoribus (n. 12.) inuentis, obtinebitur

$$\rho = \frac{BR\sin A - CR\cos A}{\sin\beta - B\sin\alpha\cos\beta + C\cos\alpha\cos\beta}$$

Ex quo sequitur, si cognita sint B et C, cum reliqua omnia sint data inueniri ~~posse~~ 1° distantiam cometae geocentricam ρ. 2do Coordinatas x, y, z: Substituto enim valore ρ in aequationibus (A) n. 12 obtinetur

(B): ..... $$x = \frac{R[\cos A\sin\beta - B\sin(\alpha - A)\cos\beta]}{\sin\beta - B\sin\alpha\cos\beta + C\cos\alpha\cos\beta}$$

$$y = \frac{R[\sin\alpha\sin\beta - C\sin(\alpha - A)\cos\beta]}{\sin\beta - B\sin\alpha\cos\beta + C\cos\alpha\cos\beta}$$

$$z = \frac{R[B\sin\alpha\sin\beta - C\cos A\sin\beta]}{\sin\beta - B\sin\alpha\cos\beta + C\cos\alpha\cos\beta}$$

et denique 3° distantiam cometae Heliocentricam, seu radium vectorem orbitae cometae

$$r = \sqrt{(x^2 + y^2 + z^2)}$$

15. Verum cum septem incognitae determinandae sint, nempe x, y, z, r, ρ, B, C: quinque autem tantum aequationes ~~dentur~~, inter has incognitas dentur, manifestum est binas adhuc aequationes ad solutionem problematis requiri. At vero si ad id animum aduertamus, quod tempora quibus cometae certos orbitarum suarum arcus describunt sint areis, quas radius vector intra id tempus verrit proportionales, facile perspicimus eo demum redire negotium, ut cum tempora per observationem haberi queant, inueniatur, si fieri possit, relatio inter radios vectores sectoresque ab iis dato tempore descriptos, seu quod idem est inter radios vectores et ipsa tempora. Hac enim relatione in genere habita, poterunt capi tot observationes cometae, quot sufficiunt ad formandas binas aequationes ~~inter radios vectores et tempora~~ ad plenam problematis solutionem requisitas.

16. Theorema.

Data parametro $Sc = p$, et anomalia $FSC = \varphi$ inuenire aream sectoris FCS.

Solutio

Ducto radio vectore Sc alteri SC infinite vicino, erit SCc elementum sectoris FCS: fiat Ce perpendicularis ad radium productum Sc, erit $Ce = r\,d\varphi$ et $d.FSC = \frac{r^2 d\varphi}{2}$, adeoque $FSC = \int\frac{r^2 d\varphi}{2}$; iam posita origine abscissarum in S et $SP = x$, ~~erit ipsaque parabola per~~ erit $PC = r\sin\varphi$, $SP = r\cos\varphi$, $PC^2 = p.FP = p(SF - SP) = p(\frac{p}{4} - r\cos\varphi)$, seu $r^2\sin\varphi^2 = p(\frac{1}{4}p - r\cos\varphi)$ unde $r = \frac{p}{2}\frac{(1-\cos\varphi)}{\sin\varphi^2} = \frac{p}{2}\left(\frac{1-\cos\varphi}{1-\cos\varphi^2}\right) = \frac{p}{2(1+\cos\varphi)}$, adeoque ob $1+\cos\varphi = 2(\cos\frac{\varphi}{2})^2$, $r = \frac{p}{4(\cos\frac{\varphi}{2})^2}$, et $\int\frac{r^2 d\varphi}{2} = \frac{p^2}{32}\int\frac{d\varphi}{(\cos\frac{\varphi}{2})^4}$.

Est vero in genere $\int\frac{du}{(\cos u)^4} = \frac{1}{3}\frac{\sin u}{(\cos u)^3} + \frac{2}{3}\frac{\sin u}{\cos u}$, (b), itaque ponendo $u = \frac{\varphi}{2}$, obtinetur

$$\int\frac{d.\frac{1}{2}\varphi}{(\cos\frac{\varphi}{2})^4} = \frac{\frac{1}{3}\sin\frac{\varphi}{2}}{(\cos\frac{\varphi}{2})^3} + \frac{2\sin\frac{\varphi}{2}}{3\cos\frac{\varphi}{2}}$$

(b) Euler Inst. Calc. Int. T. I. § 254.

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et denique $\int \frac{r^2 d\varphi}{2} = \frac{p^2 \sin\frac{\varphi}{2}}{48\,(\cos\frac{\varphi}{2})^3} + \frac{2p^2 \cdot \sin\frac{\varphi}{2}}{48\cos\frac{\varphi}{2}} = \frac{p^2 \cdot \mathrm{Tang}\frac{\varphi}{2}}{48\cos\frac{\varphi}{2}}$
$+ \frac{2p^2 \cdot \mathrm{Tang}\frac{\varphi}{2}}{48}$; est autem $(\cos\frac{\varphi}{2})^2 = \frac{1}{1+(\mathrm{tang}\frac{\varphi}{2})^2}$, quare

$$\int \frac{r^2 d\varphi}{2} = \frac{p^2}{16}\left(\mathrm{tang}\frac{\varphi}{2} + \frac{1}{3}\left(\mathrm{Tang}\frac{\varphi}{2}\right)^3\right)$$

constans nulla adiicienda est, evanescente enim anomalia φ etiam Sector evanescit.

17 Invento sectore parabolico in genere pro quacunque anomalia φ, facile invenitur area sectoris inter duos radios vectores intercepti; sit enim anomalia $C'SF = \varphi'$; $C''SF = \varphi''$ erit area sectoris

$$FC'S = \frac{p^2}{16}\left(\mathrm{tang}\frac{\varphi'}{2} + \frac{1}{3}\left(\mathrm{tang}\frac{\varphi'}{2}\right)^3\right)$$

$$FC''S = \frac{p^2}{16}\left(\mathrm{tang}\frac{\varphi''}{2} + \frac{1}{3}\left(\mathrm{tang}\frac{\varphi''}{2}\right)^3\right)$$

adeoque area sectoris inter duos radios vectores SC' et SC'' intercepti

$$C''C'S = \frac{p^2}{16}\left(\mathrm{tang}\frac{\varphi'}{2} - \mathrm{tang}\frac{\varphi''}{2} + \frac{1}{3}\left[\left(\mathrm{tang}\frac{\varphi'}{2}\right)^2 - \left(\mathrm{tang}\frac{\varphi''}{2}\right)^2\right]\right)$$

Sit $\varphi' + \varphi'' = 2\psi$, $\varphi' - \varphi'' = 2\omega$, erit 2ω angulus $C'SC''$ inter duos radios vectores comprehensus; porro ex binis his aequationibus obtinetur

$$\varphi' = \psi + \omega$$
$$\varphi'' = \psi - \omega$$

hos igitur valores φ' et φ'' in expressione sectoris $C''C'S$ substituendo obtinetur

$$C''C'S = \frac{p^2}{16}\left(\mathrm{tang}\frac{\psi+\omega}{2} - \mathrm{tang}\frac{\psi-\omega}{2} + \frac{1}{3}\left[\left(\mathrm{tang}\frac{\psi+\omega}{2}\right)^2 - \left(\mathrm{tang}\frac{\psi-\omega}{2}\right)\right.\right.$$

Ponatur $SC' = r'$ et $SC'' = r''$ erit $r' = \frac{p}{4(\cos\frac{\varphi'}{2})^2}$ et $r'' = \frac{p}{4(\cos\frac{\varphi''}{2})^2}$
adeoque $r'r'' = \frac{p^2}{16(\cos\frac{\varphi'}{2})^2(\cos\frac{\varphi''}{2})^2}$, et $p = 4\cos\frac{\varphi'}{2} \cdot \cos\frac{\varphi''}{2} \cdot \sqrt{r'r''}$
$=$

$= \frac{4(\cos\psi + \cos\omega)\sqrt{r' \cdot r''}}{2}$: porro $\sqrt{\frac{r'}{r''}} = \frac{\cos\frac{\varphi'}{2}}{\cos\frac{\varphi''}{2}} = \frac{\cos\frac{\psi+\omega}{2}}{\cos\frac{\psi-\omega}{2}}$
$= \frac{1 - \mathrm{tang}\frac{\psi}{2} \cdot \mathrm{tang}\frac{\omega}{2}}{1 + \mathrm{tang}\frac{\psi}{2} \cdot \mathrm{tang}\frac{\omega}{2}}$, adeoque $\mathrm{Tang}\frac{\psi}{2} = \frac{\sqrt{r'} - \sqrt{r''}}{(\sqrt{r'} + \sqrt{r''})\,\mathrm{tang}\frac{\omega}{2}}$.
cum igitur sit $\cos\psi = \frac{1 - (\mathrm{tang}\frac{\psi}{2})^2}{1 + (\mathrm{tang}\frac{\psi}{2})^2}$, seu substituendo valorem de $\mathrm{Tang}\frac{\psi}{2}$ et reducendo $\cos\psi = \frac{2\sqrt{r'r''} - (r'+r'')\cos\omega}{r'+r''+2\sqrt{r'r''}\cdot\cos\omega}$
habebitur $\cos\psi + \cos\omega = \frac{2\sqrt{r'r''} \cdot \sin\omega^2}{r'+r''-2\sqrt{r'r''}\cdot\cos\omega}$, et denique

$$p = \frac{4r' \cdot r'' \cdot \sin\omega^2}{r'+r''-2\sqrt{r'r''}\cdot\cos\omega}$$

Iam vero $\mathrm{Tang}\frac{\varphi'}{2} = \mathrm{tang}\frac{\psi+\omega}{2} = \frac{\mathrm{tang}\frac{\psi}{2} + \mathrm{tang}\frac{\omega}{2}}{1 - \mathrm{tang}\frac{\psi}{2} \cdot \mathrm{tang}\frac{\omega}{2}}$
$\mathrm{tang}\frac{\varphi''}{2} = \mathrm{tang}\frac{\psi-\omega}{2} = \frac{\mathrm{tang}\frac{\psi}{2} - \mathrm{tang}\frac{\omega}{2}}{1 + \mathrm{tang}\frac{\psi}{2} \cdot \mathrm{tang}\frac{\omega}{2}}$, seu substituendo valorem $\mathrm{Tang}\frac{\psi}{2}$ paulo antea inventum

$$\mathrm{Tang}\frac{\varphi'}{2} = \frac{\sqrt{r''}\cdot\cos\omega - \sqrt{r'}}{\sqrt{r''}\cdot\sin\omega}$$

$$\mathrm{Tang}\frac{\varphi''}{2} = \frac{\sqrt{r''} - \sqrt{r'}\cdot\cos\omega}{\sqrt{r'}\cdot\sin\omega}$$

Unde substituendo hos quantitatum $\mathrm{Tang}\frac{\psi}{2}$, $\mathrm{tang}\frac{\varphi'}{2}$, $\mathrm{tang}\frac{\varphi''}{2}$, atque parametri p valores obtinetur

$$C''C'S = \frac{(r'+r''+\sqrt{r'r''}\cdot\cos\omega)\sqrt{r' \cdot r''}\cdot\sin\omega}{3}, \quad (7)$$

18. Quod si iam tempus, quo cometa absolvit arcum parabolicum $C'C''$ vocetur θ' erit

$$\theta' = \frac{n\,(r'+r''+\sqrt{r'r''}\cdot\cos\omega)\sqrt{(r'+r''-2\sqrt{r'r''}\cdot\cos\omega)}}{3\sqrt{2}}$$

(7), $\sin\omega$, $\cos\omega$ signis radicalibus non afficiuntur.

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19. Si nolimus uti quantitate $\cos\omega$, in eius locum poterit substitui chorda $C''C' = k$; in triangulo enim $C''CS$ habetur per principia Trigonometriae

$$k = \sqrt{(r'^2 - 2r'r''\cos 2\omega + r''^2)}$$

adeoque $\cos 2\omega = \frac{r'^2 + r''^2 - k^2}{2r'r''}$; est autem $\cos 2\omega = 2\cos\omega^2 - 1$, quare hunc valorem substituendo, obtinetur

$$\cos\omega = \frac{\sqrt{[(r'+r'')^2 - k^2]}}{2\sqrt{r'r''}}$$

unde

$$\frac{3\theta'\sqrt{2}}{n} = (r' + r'' + \tfrac{1}{2}\sqrt{[(r'+r'')^2 - k^2]})\sqrt{(r'+r'' - \sqrt{[(r'+r'')^2 - k^2]})}$$

seu factorem $r' + r'' + \frac{1}{2}\sqrt{[(r'+r'')^2 - k^2]}$ signo radicali sup-ponendo et debita reductione instituta

$$\frac{3\theta'\sqrt{2}}{n} = \left(\frac{r'+r''+k}{2}\right)^{\frac{3}{2}} - \left(\frac{r'+r''-k}{2}\right)^{\frac{3}{2}} \ldots\ldots (D_2)$$

Formula haec, qua nihil potest excogitari elegantius, est primum a Lamberto inventa (8), sed cum metho-dus ~~quantitas~~ qua utitur minus sit directa, neque a La-Grangio evoluta habeatur, hic enim inventa illa quam ingreditur $\cos\omega$ alteri supersedet; idcirco ope-rae pretium esse duxi ex altera istam immediate deducere, nullis aliis quam Trigonometriae planae principiis suppositis.

20. Illud per se manifestum est chordam k posse exprimi per coordinatas $x', y', z', x'', y'', z''$; nam ducta $C''g$ ad $C'M'$ perpendiculari erit $C''g = M''M'$, adeoque cum sit $C''C'^2 = C''g^2 + C'g^2$, erit quoque $C''C'^2 = M''M'^2 + C'g^2$ seu

$$k = \sqrt{[(x'-x'')^2 + (y'-y'')^2 + (z'-z'')^2]}$$

(8). De insignioribus orbitae cometarum proprietatibus.

ita ut tempus quo cometa datum arcum parabolae describit non a foco, sed a differentia coordinatarum et summa radiorum vectorum dependeat.

21. Ad inveniendos nunc valores de B et C sint tria successiva cometae loca C', C'', C'''; coordinatae autem loco primae observationis cometae respondentes sint x', y', z; [...] x''', y''', z'''; valores quantitatum

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expressi (n. 7 et 10), tum loco x, y, z, [...] valores (n. 14), habebuntur 6mae aequationes, in quas nullae quantitates incognitae praeter B et C ingrediuntur, nempe

$$\theta' = f' : (B, C)$$
$$\theta'' = f'' : (B, C)$$

ex quibus proinde per methodum eliminationis poterunt determinari valores B et C.

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$$k = \sqrt{(r'^2 - 2r'r''\cos 2\omega + r''^2)}$$

adeoque $\cos 2\omega = \frac{r'^2 + r''^2 - k^2}{2r'r''}$; est autem $\cos 2\omega = 2\cos\omega^2 - 1$,

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Substitutis in expressione parametri p et areae sectoris $C''C'S$ loco $\cos\omega$ et $\sin\omega = \sqrt{(1-\cos\omega^2)}$ valoribus per r', r'' et k expressis habebitur

$$p = \frac{k^2 - (r'-r'')^2}{r'+r'' - \sqrt{[(r'+r'')^2 - k^2]}}$$

$$C''C'S = \frac{\frac{1}{2}\left(r'+r'' + \frac{1}{2}\sqrt{(r'^2+2r'r''+r''^2-k^2)}\right)\sqrt{(-r'^2+2r'r''-r''^2+k^2)}}{3}$$

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20. Illud per se manifestum est chordam k posse exprimi per coordinatas x', y', z', x'', y'', z''; nam ducta $C''g$ ad $C'M'$ perpendiculari erit $C''g = M''M'$, adeoque cum sit $C''C'^2 = C''g^2 + C'g^2$, erit quoque $C''C'^2 = M''M'^2 + C'g^2$ seu

$$k = \sqrt{[(x'-x'')^2 + (y'-y'')^2 + (z'-z'')^2]}$$

(8). De insignioribus orbitae cometarum proprietatibus.

ita ut tempus quo cometa datum arcum parabolae describit non a foco, sed a differentia coordinatarum et summa radiorum vectorum dependeat.

21. Ad inveniendos nunc valores de B et C sint tria successiva cometae loca C', C'', C'''; coordinatae autem loco primae observationis cometae respondentes sint x', y', z'; secundae x'', y'', z'', tertiae x''', y''', z'''; valores quantitatum A, α, β, R pro tempore primae observationis sint A', α', β', R'; secundae $A'', \alpha'', \beta'', R''$; tertiae $A''', \alpha''', \beta''', R'''$. manente valore quantitatum B et C perpetuo eodem, id quod per se manifestum est, cum B et C sint functiones a situ telluris et cometae prorsus independentes. Porro intervallum temporis a prima usque secundam observationem, ~~sit θ'~~, ~~ab secunda usque tertiam~~ seu tempus quo cometa ex C' pervenit in C'' sit θ', quo ex C'' in C''', seu intervallum inter secundam et tertiam observationem sit θ''. Hoc posito habebuntur binae aequationes

$$\frac{3\theta'\sqrt{2}}{n} = \left(\frac{r'+r''+k'}{2}\right)^{\frac{3}{2}} - \left(\frac{r'+r''-k'}{2}\right)^{\frac{3}{2}}$$

$$\frac{3\theta''\sqrt{2}}{n} = \left(\frac{r''+r'''+k''}{2}\right)^{\frac{3}{2}} - \left(\frac{r''+r'''-k''}{2}\right)^{\frac{3}{2}}$$

in quibus si loco r', r'', r''' et k', k'' substituantur valores earundem quantitatum per x', y', z'; x'', y'', z''; x''', y''', z''' expressi (n. 1 et 20), tum loco x', y', z'; x'', y'', z''; x''', y''', z''' valores (n. 14), habebuntur binae aequationes, in quas nullae quantitates incognitae praeter B et C ingrediuntur, nempe

$$\theta' = f'(B, C)$$

$$\theta'' = f''(B, C)$$

ex quibus proinde per methodum eliminationis poterunt determinari valores B et C.

15

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Verum haec methodus licet videatur simplicissima, attamen si substitutiones reipsa instituantur perducit ad aequationes cum altissimas, tum maxime complicatas, ita ut si quis hos calculos ingredi velit, nullum exitum reperiet. Ceterum si in subsidium vocentur coordinatae in ipso plano orbitae cometarum sumtae negotium fit longe magis expeditum, ut sequens articulus docebit.

Articulus IV

22. Repraesentet NC (fig 5) orbitam cometae, S centrum solis, N nodum orbitae, adeoque NS communem plani orbitae cometae et eclipticae intersectionem; ♈♎ est ut antea linea aequinoctiorum, SP, PM, MC tres coordinatae rectangulae per x, y, z designatae. Ducta perpendiculari CD ad DS et retentis litterarum h et i valoribus sit $SD=\sigma$ et $DC=\delta$, ob $SP=SE-EP=SE-DF$, $PM=DE+MF$ et $CM=CD \sin i$, exit

$$(E)\cdots\quad x=\sigma\cos h-\delta\cos i\sin h$$
$$y=\delta\sin h+\delta\cos i\cos h$$
$$z=\delta\sin i.$$

Quod si iam pro axe coordinatarum accipiatur quaecunque alia recta in plano orbitae posita et per centrum solis transiens SU, sintque novae coordinatae $SW=t$, $WC=u$, et distantia novi axis SU a linea nodorum, seu angulus $NSU=\mu$ habebitur

$$\sigma=t\cos\mu-u\sin\mu$$
$$\delta=t\sin\mu+u\cos\mu$$

proinde valores hos in aequationibus (E) substituendo

$$x=t(\cos h\cos\mu-\sin\mu\sin h\cos i)-u(\sin\mu\cos h+\sin h\cos i)$$
$$y=t(\sin\mu\cos h\cos i+\cos\mu\sin h)+u(\cos\mu\cos h.\cos i-\sin\mu\sin h)$$
$$z=t\sin\mu\sin i+u\cos\mu\sin i.$$

brevitatis gratia hae aequationes sequenti ratione exprimantur

$$x=at+bu,\quad y=ct+eu,\quad z=ft+gu.$$

Cum igitur quantitates a, b, c, e, f, g sint constantes, erit pro prima observatione

$$x'=at'+bu',\quad y'=ct'+eu',\quad z'=ft'+gu'$$

pro secunda

$$x''=at''+bu'',\quad y''=ct''+eu'',\quad z''=ft''+gu''$$

pro tertia

$$x'''=at'''+bu''',\quad y'''=ct'''+eu''',\quad z'''=ft'''+gu'''$$

ope primarum trium verticalium eliminentur a et b, ope secundarum c et e, et denique ope ultimarum f et g, obtinebuntur sequentes tres aequationes

$$(t'''u''-t''u''')x'-(t'''u'-t'u''')x''+(t''u'-t'u'')x'''=0$$
$$(t'''u''-t''u''')y'-(t'''u'-t'u''')y''+(t''u'-t'u'')y'''=0$$
$$(t'''u''-t''u''')z'-(t'''u'-t'u''')z''+(t''u'-t'u'')z'''=0$$

et brevitatis causa ponendo

$$t''u'-t'u''=\alpha$$
$$t'''u'-t'u'''=M$$
$$t'''u''-t''u'''=N$$

habebitur

$$\frac{N}{\alpha}x'+\frac{M}{\alpha}x''+x'''=0$$
$$\frac{N}{\alpha}y'+\frac{M}{\alpha}y''+y'''=0$$
$$\frac{N}{\alpha}z'+\frac{M}{\alpha}z''+z'''=0$$

denique substituendo loco x', y', z'; x'', y'', z''; x''', y''', z''' valores (A) n. 12 obtinebuntur tres aequationes, in quibus praeter ϱ', ϱ'', et ϱ''' nullae aliae incognitae occurrunt, ita ut per methodum eliminationis possint hi valores $\varrho'\varrho''\varrho'''$ inveniri, dummodo cogniti sint valores $\frac{N}{\alpha}$ et $\frac{M}{\alpha}$: aequationes istae sunt sequentes 16

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$$N\rho'\cos\alpha'\cos\beta' - M\rho''\cos\alpha''\cos\beta'' + L\rho'''\cos\alpha'''\cos\beta''' + NR'\cos A' - MR''\cos A'' + LR'''\cos A''' = 0$$

$$N\rho'\sin\alpha'.\cos\beta' - M\rho''\sin\alpha''\cos\beta'' + L\rho'''\sin\alpha'''\cos\beta''' + NR'\sin A' - MR''\sin A'' + LR'''\sin A''' = 0$$

$$NR'\sin\beta' - M\rho''\sin\beta'' + L\rho'''\sin\beta''' = 0.$$

quo eliminatio facilius instituatur, ponatur interea

$N\cos\alpha'\cos\beta' = A$, $M\cos\alpha''\cos\beta'' = B$, $L\cos\alpha'''\cos\beta''' = C$, $NR'\cos A' - MR''\cos A'' + LR''\cos A'$

$N\sin\alpha'\cos\beta = E$, $M\sin\alpha''\cos\beta'' = F$, $L\sin\alpha''\cos\beta''' = G$, $NR'\sin A' - MR''\sin A'' + LR'''\sin A''' = H$;

$N\sin\beta' = I$, $M\sin\beta'' = K$, $L\sin\beta'' = P$

habebuntur tres aequationes simplicius expressae

$$A\rho' - B\rho'' + C\rho''' + D = 0$$

$$E\rho' + F\rho'' + G\rho''' + H = 0$$

$$I\rho' + K\rho'' + P\rho'' = 0$$

multiplicetur secunda per indeterminatam φ, tertia per indeterminatam ψ, atque ita multiplicatae addantur primae habebitur per methodum d'Alembertii

$$A\rho' + \varphi E\rho' + \psi I\rho' - B\rho'' - \varphi F\rho'' - \psi K\rho'' + C\rho''' + \varphi G\rho''' + \psi P\rho'' + D + \varphi H = 0$$

cum iam quantitates φ et ψ sint arbitrariae, poterit illis eiusmodi valor tribui ut sit

$$B\rho'' + \varphi F\rho'' + \psi K\rho'' = 0$$

$$C\rho'' + \varphi G\rho'' + \psi P\rho'' = 0$$

quo facto erit quoque $A\rho' + \varphi E\rho' + \psi I\rho' + D + \varphi H = 0$, unde invenitur

$$\rho' = -\frac{(D + \varphi H)}{A + E\varphi + I\psi}$$

at ex primis duabus invenitur simili methodo

$$\varphi = \frac{KC - PB}{FP - KG}$$

$$\psi = \frac{-FC + BG}{FP - KG}$$

unde hos valores de φ et ψ substituendo

$$\rho' = \frac{-\left\{D + \frac{H(KC - BP)}{FP - GK}\right\}}{A + \frac{E(KC - BP)}{FP - GK} + \frac{I(BG - FC)}{FP - GK}}$$

Eadem ratione posito

$$A\rho' + \varphi E\rho' + \psi I\rho' = 0$$

$$C\rho''' + \varphi G\rho''' + \psi P\rho''' = 0$$

adeoque etiam

$$B\rho'' + \varphi F\rho'' + \psi K\rho'' - D - \varphi H = 0$$

invenitur

$$\rho'' = \frac{D + H(CI - AP) : (EP - GI)}{B + \frac{F(CI - AP)}{EP - GI} + \frac{K(AG - CE)}{EP - GI}}$$

Et denique ponendo

$$A\rho' + \varphi E\rho' + \psi I\rho' = 0$$

$$B\rho'' + \varphi F\rho'' + \psi K\rho'' = 0$$

adeoque etiam

$$C\rho''' + \varphi G\rho''' + \psi P\rho''' + D + \varphi H = 0$$

invenitur

$$\rho''' = \frac{-\left\{D + \frac{H(BI - AK)}{EK - FI}\right\}}{C + \frac{G(BI - AK)}{EK - FI} + \frac{P(AF - BE)}{EK - FI}}$$

unde valores de ρ', ρ'', ρ''' reducti erunt sequentes:

$$\rho' = -\frac{\{D(FP - GK) + H(KC - BP)\}}{A(FP - GK) + E(KC - BP) + I(BG - FC)}$$

$$\rho'' = \frac{D(EP - GI) + H(CI - AP)}{B(EP - GI) + F(CI - AP) + K(AG - CE)}$$

$$\rho''' = -\frac{\{D(EK - FE) + H(BI - AK)\}}{C(EK - FE) + G(BI - AK) + P(AF - BE)}$$ 17

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seu restitutis quantitatum A, B, C, D, E &c valoribus et notato quod sit $\sin(\alpha - A) = \sin\alpha\cos A - \sin A\cos\alpha$ habetur per actam reductione

$$\rho' = \frac{\sin\beta''\cos\beta'''[N R'\sin(\alpha'''-A') - M R''\sin(\alpha'''-A'') + \propto R'''\sin(\alpha'''-A''')]}{N[\sin(\alpha''-\alpha')\cos\beta'\cos\beta''\sin\beta''' + \sin(\alpha'''-\alpha'')\cos\beta''\cos\beta'''\sin\beta' + \sin(\alpha'-\alpha''')\cos\beta'''\cos\beta'\sin\beta'']}$$

$$- \frac{\cos\beta''\sin\beta'''[N R'\sin(\alpha''-A') - M R''\sin(\alpha''-A'') + \propto R'''\sin(\alpha''-A''')]}{N[\sin(\alpha''-\alpha')\cos\beta'\cos\beta''\cos\beta''' + \sin(\alpha'''-\alpha'')\cos\beta''\cos\beta'''\sin\beta' + \sin(\alpha'-\alpha''')\cos\beta'''\cos\beta'\sin\beta'']}$$

$$\rho'' = - \frac{\sin\beta'''\cos\beta'[N R'\sin(\alpha'-A') - M R''\sin(\alpha'-A'') + \propto R'''\sin(\alpha'-A''')]}{M[\sin(\alpha''-\alpha')\cos\beta'\cos\beta''\sin\beta''' + \sin(\alpha'''-\alpha'')\cos\beta''\cos\beta'''\sin\beta' + \sin(\alpha'-\alpha''')\cos\beta'''\cos\beta'\sin\beta'']}$$

$$+ \frac{\sin\beta'\cos\beta'''[N R'\sin(\alpha'''-A') - M R''\sin(\alpha'''-A'') + \propto R'''\sin(\alpha'''-A''')]}{M[\sin(\alpha''-\alpha')\cos\beta'\cos\beta''\sin\beta''' + \sin(\alpha'''-\alpha'')\cos\beta''\cos\beta'''\sin\beta' + \sin(\alpha'-\alpha''')\cos\beta'''\cos\beta'\sin\beta'']}$$

$$\rho''' = \frac{\sin\beta'\cos\beta''[N R'\sin(\alpha''-A') - M R''\sin(\alpha''-A'') + \propto R'''\sin(\alpha''-A''')]}{\propto[\sin(\alpha''-\alpha')\cos\beta'\cos\beta''\sin\beta''' + \sin(\alpha'''-\alpha'')\cos\beta''\cos\beta'''\sin\beta' + \sin(\alpha'-\alpha''')\cos\beta'''\cos\beta'\sin\beta'']}$$

$$- \frac{\sin\beta''\cos\beta'[N R'\sin(\alpha'-A') - M R''\sin(\alpha'-A'') + \propto R'''\sin(\alpha'-A''')]}{\propto[\sin(\alpha''-\alpha')\cos\beta'\cos\beta''\sin\beta''' + \sin(\alpha'''-\alpha'')\cos\beta''\cos\beta'''\sin\beta' + \sin(\alpha'-\alpha''')\cos\beta'''\cos\beta'\sin\beta'']}$$

Brevitatis gratia fiat

$$\gamma = \sin(\alpha''-\alpha')\cos\beta'\cos\beta''\sin\beta''' + \sin(\alpha'''-\alpha'')\cos\beta''\cos\beta'''\sin\beta' + \sin(\alpha'-\alpha''')\cos\beta'''\cos\beta'\sin\beta''$$

$$\Pi' = N R'\sin(\alpha'-A') - M R''\sin(\alpha'-A'') + \propto R'''\sin(\alpha'-A''')$$

$$\Pi'' = N R'\sin(\alpha''-A') - M R''\sin(\alpha''-A'') + \propto R'''\sin(\alpha''-A''')$$

$$\Pi''' = N R'\sin(\alpha'''-A) - M R''\sin(\alpha'''-A'') + \propto R'''\sin(\alpha'''-A''')$$

erit que

(7).... $$\rho' = \frac{\sin\beta''\cos\beta'''\Pi''' - \sin\beta'''\cos\beta''\Pi''}{N\gamma}$$

$$\rho'' = \frac{\sin\beta'\cos\beta'''\Pi''' + \sin\beta'''\cos\beta'\Pi'}{M\gamma}$$

$$\rho''' = \frac{\sin\beta'\cos\beta''\Pi'' - \sin\beta''\cos\beta'\Pi'}{\propto\gamma}$$

23. Cognitis M, N, $\propto$, adeoque etiam distantiis cometae geocentricis ρ', ρ'', ρ''' quae ab illis dependent, innotescent etiam distantiae cometae heliocentricae, seu radii vectores r', r'' r'''; cum enim sit in genere $r^2 = x^2 + y^2 + z^2 = R^2 + 2\rho R\cos(\alpha - A)\cos\beta + \rho^2$, (n. 11 et 12), erit quoque

$$r'^2 = R'^2 + 2\rho' R'\cos(\alpha'-A')\cos\beta' + \rho'^2$$

$$r''^2 = R''^2 + 2\rho'' R''\cos(\alpha''-A'')\cos\beta'' + \rho''^2$$

$$r'''^3 = R'''^3 + 2\rho''' R'''\cos(\alpha'''-A''')\cos\beta''' + \rho'''^2$$

24. Tota igitur difficultas in eo versatur, ut inveniantur valores M, N, $\propto$, seu rationes $\frac{M}{\propto}$ et $\frac{N}{\propto}$ (n 22), quod sane nullum negotium facesseret, si supponere liceret, cometam saltem per exiguum arcum $C'C''C'''$ (Fig 6) ferri motu rectilineo uniformi: nam posito quod exigua orbitae cometae portio $C'C''C'''$ sit linea recta, ob triangula $C'C''A$ et $C''C'''B$ similia haberetur ~~&c~~:

$$C'C'' : C'A = C''C''' : C''B$$

$$C'C'' : C''A = C''C''' : C'''B$$

seu ob hypothesim motus uniformis, adeoque spatia $C'C''$ et $C''C'''$ temporibus θ' et θ'' proportionalia

$$\theta' : u'-u'' = \theta'' : u''-u'''$$

$$\theta' : t''-t' = \theta'' : t'''-t''$$

unde

$$u''' = \frac{(\theta'+\theta'')u'' - \theta''u'}{\theta'}$$

$$t''' = \frac{(\theta'+\theta'')t'' - \theta''t'}{\theta'}$$

quibus valoribus in M et N (n. 22) substitutis, obtinetur

$$M = \frac{(\theta'+\theta'')(t''u'-t'u'')}{\theta'} = \frac{\propto(\theta'+\theta'')}{\theta'}$$

$$N = \frac{\theta''(t''u'-t'u'')}{\theta'} = \frac{\propto\theta''}{\theta'}$$

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proinde

$$\frac{M}{\alpha}=\frac{\theta'+\theta''}{\theta'} \quad \text{et} \quad \frac{N}{\alpha}=\frac{\theta''}{\theta'}.$$

At vero haec methodus insigni vitio laborat, propterea quod supponatur celeritates elementares iuxta directiones t et u vi acceleratrici debitas posse exprimi aequationibus $d\frac{dt}{d\theta}=0$, et $d\frac{du}{d\theta}=0$ in genere, si supponatur motus fieri in arcu infinite parvo, cum tamen ostensum sit n. 2 dictas celeritates exprimi aequationibus $d\frac{dt}{d\theta}+\frac{Ft\,d\theta}{r^3}=0$ et $d\frac{du}{d\theta}+\frac{Fu\,d\theta}{r^3}=0$, sitque praeterea manifestum terminos $\frac{Ft\,d\theta}{r^3}$ et $\frac{Fu\,d\theta}{r^3}$ non posse negligi prae terminis $d\frac{dt}{d\theta}$, et $d\frac{du}{d\theta}$, cum illi aeque ac hi designent fluxionis ordinis primi; Ceterum vitiosas has celeritatum elementarium expressiones tacite in methodo supponi, palam fiet, si in expressionibus t''' et u''' fiat $\theta'=d\theta$ et $\theta''=d\theta+dd\theta$, quo facto erit $t'=t$, $t''=t+dt$, $t'''=t+2dt+ddt$, uti et $u'=u$, $u''=u+du$; $u'''=u+2du+ddu$; ~~adeoque~~ cum enim aequationes $t'''=\frac{(\theta'+\theta'')t''-\theta''t'}{\theta'}$ et $u'''=\frac{(\theta'+\theta'')u''-\theta''u'}{\theta'}$ reducantur ad has $\frac{t'''-t''}{\theta''}-\cancel{\theta}\frac{t''-t'}{\theta'}=0$ et $\frac{u'''-u''}{\theta''}-\frac{u''-u'}{\theta'}=0$, substitutis dictis valoribus de θ' et θ'' deque t', t'', t''', u', u'', u''' obtinebitur $\cancel{t}$ $\frac{dt+ddt}{d\theta+dd\theta}-\frac{dt}{d\theta}=0$, $\frac{du+ddu}{d\theta+dd\theta}-\frac{du}{d\theta}=0$, seu $d\frac{dt}{d\theta}=0$ et $d\frac{du}{d\theta}=0$, id quod manifesto indicio est, terminos $\frac{Ft\,d\theta}{r^3}$ et $\frac{Fu\,d\theta}{r^3}$ haberi pro nullis relate ad terminos $d\frac{dt}{d\theta}$ et $d\frac{du}{d\theta}$.

24. Cum igitur hypothesis motus rectilinei uniformis nequeat habere locum, seu quod idem est coordinatae orbitae cometae nequeant haberi pro functionibus linearibus temporis, sit tempus secundae observationis a quodam termino fixo tanquam epocha computatum $=\theta$, erit tempus primae observationis $=\theta-\theta'$; tertiae observationis $=\theta+\theta''$; Quod si igitur assumatur coordinatas orbitae cometae esse qualescunque functiones temporis seu esse $t''=f:(\theta)$, $u''=\varphi:(\theta)$, manifestum est abeunte θ in $\theta-\theta'$, abire t'' in t' et u'' in u'; abeunte vero θ in $\theta+\theta''$ abire t'' in t''' et u'' in u'''; est igitur per formulam Taylorianam

$$t'=t''-\frac{\theta'dt''}{d\theta}+\frac{\theta'^2d^2t''}{2d\theta^2}-\&c$$

$$u'=u''-\frac{\theta'du''}{d\theta}+\frac{\theta'^2d^2u''}{2d\theta^2}-\&c$$

$$t'''=t''+\frac{\theta''dt''}{d\theta}+\frac{\theta''^2d^2t''}{2d\theta^2}+\&c$$

$$u'''=u''+\frac{\theta''du''}{d\theta}+\frac{\theta''^2d^2u''}{2d\theta^2}+\&c$$

Sed sumto $d\theta$ pro constante, habetur per n. 2

$$\frac{d^2t''}{d\theta^2}=-\frac{Ft''}{r''^3}$$

$$\frac{d^2u''}{d\theta^2}=-\frac{Fu''}{r''^3}$$

quare hos valores substituendo obtinetur

$$t'=t''-\frac{\theta'dt''}{d\theta}-\frac{\theta'^2Ft''}{2r''^3}\ \&c$$

$$u'=u''-\frac{\theta'du''}{d\theta}-\frac{\theta'^2Fu''}{2r''^3}\ \&c$$

$$t'''=t''+\frac{\theta''dt''}{d\theta}-\frac{\theta''^2Ft''}{2r''^3}\ \&c$$

$$u'''=u''+\frac{\theta''du''}{d\theta}-\frac{\theta''^2Fu''}{2r''^3}\ \&c$$

25. Si iam isti valores t', u', t''', u''' in expressionibus L, M, N, (n 22) substituantur, et termini in quibus altiores quas secundae potestates quantitatum θ', θ'' occurrunt omittantur, habebitur: 19

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$$L = \frac{u''dt'' - t''du''}{d\theta}\,\theta'$$

$$M = \frac{u''dt'' - t''du''}{d\theta}(\theta'+\theta'')\left(1 - \frac{7\theta'\theta''}{2r''^3}\right)$$

$$N = \frac{u''dt'' - t''du''}{d\theta}\,\theta''$$

qui iterum valores in Π', Π'', Π''' substituti dabunt

$$\Pi' = R'\sin(\alpha''-A')\cdot\frac{u''dt''-t''du''}{d\theta}\theta''$$
$$- R''\sin(\alpha''-A'')\cdot\frac{u''dt''-t''du''}{d\theta}(\theta'+\theta'')\left(1-\frac{7\theta'\theta''}{2r''^3}\right)$$
$$+ R'''\sin(\alpha'-A''')\cdot\frac{u''dt''-t''du''}{d\theta}\theta'$$

$$\Pi'' = R'\sin(\alpha''-A')\cdot\frac{u''dt''-t''du''}{d\theta}\theta''$$
$$- R''\sin(\alpha''-A'')\cdot\frac{u''dt''-t''du''}{d\theta}(\theta'+\theta'')\left(1-\frac{7\theta'\theta''}{2r''^3}\right)$$
$$+ R'''\sin(\alpha''-A''')\cdot\frac{u''dt''-t''du''}{d\theta}\theta'$$

$$\Pi''' = R'\sin(\alpha'''-A')\cdot\frac{u''dt''-t''du''}{d\theta}\theta''$$
$$- R''\sin(\alpha'''-A'')\cdot\frac{u''dt''-t''du''}{d\theta}(\theta'+\theta'')\left(1-\frac{7\theta'\theta''}{2r''^3}\right)$$
$$+ R'''\sin(\alpha'''-A''')\cdot\frac{u''dt''-t''du''}{d\theta}\theta'$$

Dein ponendo maioris simplicitatis gratia

$$w' = R'\theta''\sin(\alpha'-A') - R''(\theta'+\theta'')\sin(\alpha'-A'') + R'''\theta'\sin(\alpha'-A''')$$
$$w'' = R'\theta''\sin(\alpha''-A') - R''(\theta'+\theta'')\sin(\alpha''-A'') + R'''\theta'\sin(\alpha''-A''')$$
$$w''' = R'\theta''\sin(\alpha'''-A') - R''(\theta'+\theta'')\sin(\alpha'''-A'') + R'''\theta'\sin(\alpha'''-A''')$$

ut habeatur

$$\Pi' = \left[w' + \frac{7(\theta'+\theta'')\theta'\theta''R''\sin(\alpha'-A'')}{2r''^3}\right]\cdot\frac{u''dt''-t''du''}{d\theta}$$
$$\Pi'' = \left[w'' + \frac{7(\theta'+\theta'')\theta'\theta''R''\sin(\alpha''-A'')}{2r''^3}\right]\cdot\frac{u''dt''-t''du''}{d\theta}$$
$$\Pi''' = \left[w''' + \frac{7(\theta'+\theta'')\theta'\theta''R''\sin(\alpha'''-A'')}{2r''^3}\right]\cdot\frac{u''dt''-t''du''}{d\theta}$$

26. Ceterum hae expressiones quantitatum Π', Π'', Π''' simpliciores adhuc reddi possunt; cum enim orbita terrae sit proxime circularis, si sumantur tempora θ', θ'', quod semper hactenus factum est, pro admodum exiguis, erunt 1mo radii orbitae terrae R', R'', R''' proxime aequales, 2do ~~celeritatem angulares~~ ob motum in hac hypothesi aequabilem, differentiae longitudinum $A''-A'$, $A'''-A''$ temporibus θ' et θ'' proportionales. Cum igitur in hoc casu posita celeritate angulari terrae $= m$, sit $A''-A' : m = \theta' : 1$, et $A'''-A'' : m = \theta'' : 1$ erit $A''-A' = m\theta'$ et $A'''-A'' = m\theta''$, hinc

$$A' = A'' - m\theta'$$
$$A''' = A'' + m\theta''$$

Hos igitur valores in w', w'', w''' substituendo obtinebitur

$$w' = R''[\theta''\sin(\alpha'-A''+m\theta') - (\theta'+\theta'')\sin(\alpha'-A'') + \theta'\sin(\alpha'-A''-m\theta'')]$$
$$w'' = R''[\theta''\sin(\alpha''-A''+m\theta') - (\theta'+\theta'')\sin(\alpha''-A'') + \theta'\sin(\alpha''-A''-m\theta'')]$$
$$w''' = R''[\theta''\sin(\alpha'''-A''+m\theta') - (\theta'+\theta'')\sin(\alpha'''-A'') + \theta'\sin(\alpha'''-A''-m\theta'')]$$

Sed ob quantitates θ' et θ'' admodum parvas est

$$\sin(\alpha'-A''+m\theta') = \sin(\alpha'-A'') + m\theta'\cos(\alpha'-A'') - \frac{m^2\theta'^2}{2}\sin(\alpha'-A'') - \frac{m^3\theta'^3}{6}\cos(\alpha'-A'') + \text{etc.}$$

$$\sin(\alpha'-A''-m\theta'') = \sin(\alpha'-A'') - m\theta''\cos(\alpha'-A'') - \frac{m^2\theta''^2}{2}\sin(\alpha'-A'') + \frac{m^3\theta''^3}{6}\cos(\alpha'-A'') - \text{etc.}$$

est enim in genere posito quod ω sit quantitas valde parva $\sin(x \pm \omega) = \sin x \pm \omega\cos x - \frac{\omega^2\sin x}{2} \mp \frac{\omega^3\cos x}{6} \mp \text{etc.}$ Euler Instit. Calc. Differ. §. 95. His igitur ~~q~~ valoribus in w', w'', w''' substitutis et quantitatibus tertio ordine altioribus neglectis fiet

$$w' = -\frac{m^2(\theta'+\theta'')\theta'\theta''}{2}R''\cdot\sin(\alpha'-A'')$$
$$w'' = -\frac{m^2(\theta'+\theta'')\theta'\theta''}{2}R''\sin(\alpha''-A'')$$
$$w''' = -\frac{m^2(\theta'+\theta'')\theta'\theta''}{2}R''\sin(\alpha'''-A'')$$

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adeoque

$$\Pi' = -\frac{(\theta'+\theta'')\theta'\theta''R''}{2}\left(m^2-\frac{F}{r''^3}\right)\sin(\alpha'-A'').\ \frac{u''dt''-t''du''}{d\theta}$$

$$\Pi'' = -\frac{(\theta'+\theta'')\theta'\theta''R''}{2}\left(m^2-\frac{F}{r''^3}\right)\sin(\alpha''-A'').\ \frac{u''dt''-t''du''}{d\theta}$$

$$\Pi''' = -(\theta'+\theta'')\theta'\theta''R''\left(m^2-\frac{F}{r''^3}\right)\sin(\alpha'''-A'').\ \frac{u''dt''-t''du''}{d\theta}$$

In expressionibus w', w'', w''' nequeunt termini ordinis tertii ommitti propterea, quod in expressionibus primitivis ipsarum Π', Π'', Π''' n. 25, praeter w', w'', w''' occurrant quantitates $(\theta'+\theta'')\theta'\theta''$ quae et ipsae sunt ordinis tertii.

27. Inventis valoribus quantitatum $\alpha, M, N; \Pi', \Pi'', \Pi'''$ habebuntur etiam ex aequationibus (F) n. 22 tres distantiae geocentricae $\varrho'\ \varrho''\ \varrho'''$, nimirum

$$\varrho' = \frac{(\theta'+\theta'')\theta'}{2\gamma}\mu' R''\left(\frac{F}{r''^3}-m^2\right)$$

$$\varrho'' = -\frac{\theta'\theta''\mu''.\,R''\left(\frac{F}{r''^3}-m^2\right)}{2\gamma\left(1+\frac{F\theta'\theta''}{2r''^3}\right)}$$

$$\varrho''' = \frac{(\theta'+\theta'')\theta''\mu'''R''}{2\gamma}\left(\frac{F}{r''^3}-m^2\right)$$

posita quantitate nota

$\sin\beta''\cos\beta'''\sin(\alpha'''-A'') - \sin\beta'''\cos\beta''\sin(\alpha''-A'') = \mu$

$\sin\beta'''\cos\beta'\sin(\alpha'-A'') + \sin\beta'\cos\beta'''\sin(\alpha'''-A'') = \mu$

$\sin\beta'\cos\beta''\sin(\alpha''-A'') - \sin\beta''\cos\beta'\sin(\alpha'-A'') = \mu$

quarum secunda ob $\frac{F\theta'\theta''}{2r''^3}$ relate a t evanescentem reducitur ad hanc

$$(G)\ldots\quad \varrho'' = \frac{\theta'\theta''\mu''.\,R''\left(m^2-\frac{F}{r''^3}\right)}{2\gamma}$$

Est vero $r''^2 = R''^2 + 2\varrho''R''\cos(\alpha''-A'')\cos\beta'' + \varrho''^2$, quare si hic valor in aequatione (G) substituat obtinebitur aequatio in qua praeter ϱ'' nulla incognita occurrit, et ex qua proinde poterit determinari ϱ''. Ad hanc quantitatis r'' eliminationem reipsa instituendam, ~~quaeratur primum valor~~ ~~r''^3 ex aequatione (G) ponitur~~ ponatur primum in aequatione (G)

$\frac{\theta'\theta''\mu''R''}{2\gamma} = \lambda$, ut habeatur $\varrho'' = \lambda\left(m^2-\frac{F}{r''^3}\right)$, obtinebitur hinc $(\varrho''-\lambda m^2)r''^3 = -\lambda F$, et sumendo quadrata $(\varrho''-\lambda m^2)^2 r''^6 = \lambda^2F^2$ seu $(\varrho''-\lambda m^2)^2\left(R''^2+2\varrho''R''\cos(\alpha''-A'')\cos\beta''+\varrho''^2\right)^3 - \lambda^2F^2 = 0$; haec evoluta iuxtaque ϱ ordinata abit in sequentem

$$(H)\ldots\quad \varrho''^8 + \left.\begin{matrix}3v\\-2\lambda m^2\end{matrix}\right\}\varrho''^7 + \left.\begin{matrix}3R''^2\\+3v^2\\-6\lambda m^2v\\+\lambda^2m^4\end{matrix}\right\}\varrho''^6 + \left.\begin{matrix}6R''^2v\\+3R''^4v\\+v^3\\-6\lambda m^2v^2\\-6R''^2\lambda m^2\\+3\lambda^2m^4v\end{matrix}\right\}\varrho''^5 + \left.\begin{matrix}3R''^2v^2\\-2R''^6\lambda m^2\\-2\lambda m^2v^3\\-6R''^4v\lambda m^2\\-12R''^2\lambda m^2v\\+3v^2\lambda^2m^4\\+3R''^2\lambda^2m^4\end{matrix}\right\}\varrho''^4 + \left.\begin{matrix}3R''^4v\\-6R''^2v^2\lambda m^2\\+\lambda^2m^4v^3\\+3R''^4\lambda^2m^4v\\+6R''^2\lambda^2m^4v\end{matrix}\right\}\varrho''^3 + \left.\begin{matrix}R''^6\\-6R''^4v\lambda m^2\\+3R''^2v^2\lambda^2m^4\end{matrix}\right\}\varrho''^2 + 3R''^4\lambda^2m^4v\varrho''$$

$$+\lambda^2\left(R''^6m^4-F^2\right) = 0$$

denotante v quantitatem $2R''\cos(\alpha''-A'')\cos\beta''$. Iam vero m est celeritas angularis ~~in~~ corporis, cuius distantia a centro motus est R'' eius igitur celeritas vera est $= mR''$, quare cum vis centrifuga aequetur quadrato celeritatis verae divisae per radium, erit in hoc casu vis centrifuga $= \frac{m^2R''^2}{R''} = m^2R''$; ~~cum igitur~~ at vero cum orbita terrae sit proxime circularis, erit pro ea vis centrifuga in distantia R'' vi attractrici in eadem distantia proxime aequalis, quae igitur cum sit $= \frac{F}{R''^2}$ erit $\frac{F}{R''^2} = m^2R''$, et $F = m^2R''^3$, quare $F^2 = m^4R''^6$, et ultimus aequationis superioris terminus $\lambda^2(R''^6m^4-F^2) = 0$; unde consequitur aequationem (H), quae est octavi gradus, ad gradum septimum per naturam problematis deprimi, ~~unamque~~ radicem ~~unam~~ habere necessario ~~esse~~ realem.

Cum iam eadem plane ratione inveniri possint distantiae ϱ' et ϱ''', et ~~habent~~ habitis his regrediendo ad aequationes superiores promum sit omnia elementa orbitae cometarum determinare, his iam non immorabor, sed eum casum evolvam qui rei naturae convenientior est, hoc est in quo

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tempora θ', et θ'' non sunt fluxiones, quemadmodum hactenus suppositum erat, sed magnitudinis finitae, et qui proprie debet ~~ope~~ scopus ~~~~ huius argumenti.

Articulus V

28. Quemadmodum in articulo praecedenti, ante omnia valores quantitatum α, m, n determinavimus, ita etiam in praesenti ab iisdem quantitatibus initium faciemus: Dico igitur valores quantitatum α, m, n non esse aliud, quam duplos ~~tert~~ areas sectorum triangularium inter binos radios vectores et chordam ~~t~~ singula bina cometae loca iungentem; seu esse $\alpha = 2C''C'S$ (fig 4), $m = 2C'''C'S$, $n = 2C'''C''S$. Sufficiet unicum ostendere, reliqua per se intelligentur.

Posito $C'S = r'$, $C''S = r''$, chorda $C''C' = k$, et angulo $C''SC' = \varphi$, erit demissa $C'N$ ad $C''S$ perpendiculari, $C'N = r'\sin\varphi$ et area sectoris triangularis $C''C'S$ quam vocabo $\Delta = \frac{r'r''\sin\varphi}{2}$. est vero iuxta principia trigonometriae planae

$$\sin\varphi = \frac{\sqrt{(r'^2+2r'r''+r''^2-k^2)(-r'^2+2r'r''-r''^2+k^2)}}{2r'r''}$$

quamobrem

$$\Delta = \tfrac{1}{4}\sqrt{(r'^2+2r'r''+r''^2-k^2)(-r'^2+2r'r''-r''^2+k^2)}$$

Porro $r'^2 = t'^2+u'^2$, $r''^2 = t''^2+u''^2$, $k^2 = (t'-t'')^2+(u''-u')^2$, ~~hoc igit~~ hos igitur valores substituendo obtinetur

$$\Delta = \tfrac{1}{4}\sqrt{[2\sqrt{(t'^2+u'^2)(t''^2+u''^2)}+2t't''+2u'u''][2\sqrt{(t'^2+u'^2)(t''^2+u''^2)} - 2t't''-2u'u'']}$$

seu reipsa multiplicando

$$\Delta = \tfrac{1}{4}\sqrt{(4t'^2t''^2-8t't''u'u''+4t''^2u'^2)}$$

adeoque

$$\Delta = \frac{t'u''-t''u}{2}\,..$$

29. Cum igitur sit $\alpha = 2\Delta = \tfrac{1}{2}\sqrt{[(r'+r'')^2-k^2][k^2-(r'-r'')^2]}$

erit $\frac{\alpha}{\sqrt{p}} = \tfrac{1}{2}\sqrt{[(r'+r'')^2-k^2][r'+r''-\sqrt{[(r'+r'')^2-k^2)}]}$: at per n.19

habetur $\frac{3\theta'\sqrt{2}}{n(r'+r''+\frac{1}{2}\sqrt{[(r'+r'')^2-k^2]}} = \sqrt{[(r'+r''-\sqrt{(r'+r'')^2-k^2)}]}$, quare

$$\frac{\alpha}{\sqrt{p}} = \frac{3\theta'\sqrt{2}.\sqrt{[(r'+r'')^2-k^2]}}{2n(r'+r''+\frac{1}{2}\sqrt{[(r'+r'')^2-k^2]}}$$

seu

$$\frac{2n\alpha}{3\sqrt{2p}} = \frac{\theta'.\frac{1}{2}\sqrt{[(r'+r'')^2-k^2]}}{r'+r''+\frac{1}{2}\sqrt{[(r'+r'')^2-k^2]}}$$

Iam nunc oportet exprimere α per quantitates r' et r'' id quod facile fieri posset, si ex aequatione

$$\theta' = \frac{n(r'+r''+\sqrt{2r'r''.\cos\omega})\sqrt{(r'+r''-2\sqrt{r'r''.\cos\omega})}}{3\sqrt{2}}, \quad (n.18)$$

et $\quad k = \sqrt{(r'^2-2r'r''\cos 2\omega+r''^2)} \quad (n.19)$

eliminetur $\cos\omega$, hac enim ratione perveniretur ad valorem k per quantitates notas, et r', r'' expressum, quo in aequatione praecedente ad α substituto obtineretur quantitas α per r' et r'', expressa: ~~~~ Hac eliminatione ipsius $\cos\omega$ reipsa instituta perveni ad aequationem sexti gradus huius formae $k^6+\alpha k^4+\beta k^2+\gamma = 0$. quae cum ad cubicam reducatur, eius omnes radices methodo Cardani exhiberi possunt: verum illius usum in determinandis valoribus α, m, n non vacat exponere, alias id negotium resumemus; ceterum in ~~ea~~ novam methodum inveniendi elementa orbitae cometarum latere, facile perspici potest.

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30. His igitur tantum per occasionem dictis, fiat brevitatis gratia $T = \frac{\frac{1}{2}\sqrt{[(r'+r'')^2 - k^2]}}{r'+r''+\frac{1}{2}\sqrt{[(r'+r'')^2-k^2]}}$ et $r'+r'' = 2\sigma$

erit $\frac{2n\mathcal{L}}{3\sqrt{2p}} = \theta' T$, et $\mathcal{L} = \frac{3\sqrt{2p}}{2n}.\theta' T$; jam vero ex praecedente substitutione invenitur $\frac{1}{2}\sqrt{[(r'+r'')^2 - k^2]} = \frac{2\sigma T}{1-T}$, quare si hi valores in aequationem (D I) n. 19 introducantur, obtinebitur

$$\frac{3\theta'\sqrt{2}}{n} = (2\sigma)^{\frac{3}{2}}.\frac{\sqrt{(1-3T)}}{(1-T)^{\frac{3}{2}}}$$

quae iterum posito $T = 1 - \frac{2v}{3}$ abibit in sequentem

$$v = 1 + \frac{\theta'^2 v^3}{6n^2\sigma^3}$$

At vero ostendit La Grangius aequationis cuiusvis formae $a - bx + cx^n$, radicem unam esse

$$1 + \frac{ca^{n-1}}{b^n} + \frac{2n\,c^2a^{2n-2}}{2b^{2n}} + \frac{3n.(3n-1)\,c^3a^{3n-3}}{2.3\,b^{3n}} + \&c \quad (9)$$

quare posito $a = 1$, $b = 1$ ~~et~~ $c = \frac{\theta'^2}{6n^2\sigma^3}$ et $n = 3$, habebitur

$$v = 1 + \frac{\theta'^2}{6n^2\sigma^3} + \frac{\theta'^4}{12.n^4\sigma^6} + \frac{\theta'^6}{18n^6\sigma^9} + \&c$$

seu ob $\sigma = \frac{r'+r''}{2}$

$$v = 1 + \frac{\theta'^2}{6n^2(\frac{r'+r''}{2})^3} + \frac{\theta'^4}{12.n^4(\frac{r'+r''}{2})^6} + \frac{\theta'^6}{18n^6(\frac{r'+r''}{2})^9} + \&c$$

adeoque negligendo terminos in quibus θ' ad altiores ~~qu~~ secunda~~m~~ potestates assurgit, habebitur valor quantitatis T approximatus

$$T = 1 - \frac{2}{3}\left(1 + \frac{\theta'^2}{6n^2(\frac{r'+r''}{2})^3}\right)$$

et denique

(9) La Grange. Memoires de l'Academie de Berlin T. XXIV: aus Eulers Anleitung in die Analysis des Unendl. von Michelsen übersetzt 5te Abh. § 34.

$$\mathcal{L} = \left[\theta' - \frac{\theta'^3}{3n^2(\frac{r'+r''}{2})^3}\right]\frac{\sqrt{2p}}{2n}$$

eademque plane ratione

$$\mathcal{M} = \left[\theta' + \theta'' - \frac{(\theta'+\theta'')^3}{3n^2(\frac{r'+r'''}{2})^3}\right]\frac{\sqrt{2p}}{2n}$$

$$\mathcal{N} = \left[\theta'' - \frac{\theta''^3}{3n^2(\frac{r''+r'''}{2})^3}\right]\frac{\sqrt{2p}}{2n}$$

Hinc $\frac{\mathcal{N}}{\mathcal{L}} = \left(\theta'' - \frac{\theta''^3}{3n^2(\frac{r''+r'''}{2})^3}\right) : \left(\theta' - \frac{\theta'^3}{3n^2(\frac{r'+r''}{2})^3}\right)$

$$\frac{\mathcal{M}}{\mathcal{L}} = \left(\theta' + \theta'' - \frac{(\theta'+\theta'')^3}{3n^2(\frac{r'+r'''}{2})^3}\right) : \left(\theta' - \frac{\theta'^3}{3n^2(r'+r'')^3}\right)$$

Si quantitatum θ', θ'' potestates prima altiores negligantur, habebitur $\frac{\mathcal{N}}{\mathcal{L}} = \frac{\theta''}{\theta'}$ et $\frac{\mathcal{M}}{\mathcal{L}} = \frac{\theta'+\theta''}{\theta'}$, qui est casus motus uniformis rectilinei, et cuius defectum in praecedenti articulo comprobavimus.

30 Inventis $\mathcal{L}$, $\mathcal{M}$, $\mathcal{N}$ et discriminis gratia ~~vocentur~~ ~~R'R''R'''~~ posito $\mathcal{L} = \Lambda'$, $\mathcal{N} = \Lambda''$, et $\mathcal{M} = \Lambda'''$ habebitur

$$\Pi' = \Lambda'' R' \sin(\alpha' - A') - \Lambda''' R'' \sin(\alpha' - A'') + \Lambda' R''' \sin(\alpha' - A''') = \Delta'$$

$$\Pi'' = \Lambda'' R' \sin(\alpha'' - A') - \Lambda''' R'' \sin(\alpha'' - A'') + \Lambda' R''' \sin(\alpha'' - A''') = \Delta''$$

$$\Pi'' = \Lambda'' R \sin(\alpha''' - A') - \Lambda''' R'' \sin(\alpha''' - A'') + \Lambda' R''' \sin(\alpha''' - A''') = \Delta'''$$

adeoque

$$\varsigma' = \frac{\sin\beta''\cos\beta'''\Delta''' - \sin\beta'''\cos\beta''\Delta''}{\Lambda''\gamma}$$

$$\varsigma'' = \frac{\sin\beta'\cos\beta''\Delta''' - \sin\beta'''\cos\beta'\Delta'}{\Lambda'''\gamma}$$

$$\varsigma''' = \frac{\sin\beta'\cos\beta''\Delta'' - \sin\beta''\cos\beta'\Delta'}{\Lambda'\gamma}$$

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Iam vero posito sicut in n. 16 tam R''' quam $R'=R''$ et $A'=A''-m\theta'$, $A'''=A''+m\theta''$ obtinebitur

$$\Delta'=\Lambda''R''\sin(\alpha'-A''+m\theta')-\Lambda'''R''\sin(\alpha'-A'')+\Lambda'R''\sin(\alpha'-A''-m\theta'')$$

$$\Delta''=\Lambda''R''\sin(\alpha''-A''+m\theta')-\Lambda'''R''\sin(\alpha''-A'')+\Lambda'R''\sin(\alpha''-A''-m\theta'')$$

$$\Delta'''=\Lambda''R''\sin(\alpha'''-A''+m\theta')-\Lambda'''R''\sin(\alpha'''-A'')+\Lambda'R''\sin(\alpha'''-A''-m\theta'')$$

quare cum sit in genere posito θ valde parvo

$$\sin(\alpha-A+m\theta)=\sin(\alpha-A)+m\theta\cos(\alpha-A)-\frac{m^2\theta^2}{2}\sin(\alpha-A)-\frac{m^3\theta^3}{6}\cos(\alpha-A)+\text{etc}$$

$$\sin(\alpha-A-m\theta)=\sin(\alpha-A)-m\theta\cos(\alpha-A)-\frac{m^2\theta^2}{2}\sin(\alpha-A)+\frac{m^3\theta^3}{6}\cos(\alpha-A)-\text{etc}$$

substitutis his valoribus et neglectis terminis in quibus θ' excedit dimensionem quartam, propterea quod Λ', Λ'', Λ''' usque ad dimensionem etiam quartam habeantur exacte, obtinetur

$$\Delta'=\frac{-(\theta'+\theta'')\theta'\theta''\sin(\alpha'-A'')}{2R''^3}\cdot\frac{V2p}{2n}-\frac{(\theta'^2-\theta''^2)\theta'\theta''\cos(\alpha'-A'')}{6R''^5}\cdot\frac{V2p}{2n}$$

$$-\left(\frac{\theta'^3}{(r'+r'')^3}-\frac{(\theta'+\theta'')^3}{(r'+r''')^3}+\frac{\theta''^3}{(r''+r''')^3}\right)\sin(\alpha'-A'')R''\cdot\frac{4V2p}{3n^3}$$

$$-\left(\frac{\theta''^2}{(r''+r''')^3}-\frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''\cos(\alpha'-A'')\cdot\frac{4V2p}{3R''n^3}$$

$$\Delta''=-\frac{(\theta'+\theta'')\theta'\theta''\sin(\alpha''-A'')}{2R''^3}\cdot\frac{V2p}{2n}-\frac{(\theta'^2-\theta''^2)\theta'\theta''\cos(\alpha''-A'')}{6R''^5}\cdot\frac{V2p}{2n}$$

$$-\left(\frac{\theta'^3}{(r'+r'')^3}-\frac{(\theta'+\theta'')^3}{(r'+r''')^3}+\frac{\theta''^3}{(r''+r''')^3}\right)\sin(\alpha''-A'')R''\cdot\frac{4V2p}{3n^3}$$

$$-\left(\frac{\theta''^2}{(r''+r''')^3}-\frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''\cos(\alpha''-A'')\cdot\frac{4V2p}{3R''n^3}$$

$$\Delta'''=-\frac{(\theta'+\theta'')\theta'\theta''\sin(\alpha'''-A'')}{2R''^3}\cdot\frac{V2p}{2n}-\frac{(\theta'^2-\theta''^2)\theta'\theta''\cos(\alpha'''-A'')}{6R''^5}\cdot\frac{V2p}{2n}$$

$$-\left(\frac{\theta'^3}{(r'+r'')^3}-\frac{(\theta'+\theta'')^3}{(r'+r''')^3}\right)\theta'\theta''\sin(\alpha'''-A'')\cdot R''\cdot\frac{4V2p}{3n^3}$$

$$-\left(\frac{\theta''^2}{(r''+r''')^3}-\frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''\cos(\alpha'''-A'')\cdot\frac{4V2p}{3R''n^3}$$

Hinc posito $\Lambda'=\frac{T'V2p}{2n}$, $\Lambda''=\frac{T''V2p}{2n}$, $\Lambda'''=\frac{T'''V2p}{2n}$ et substitutis valoribus quantitatum Δ', Δ'', Δ''' uti et Λ', Λ'', Λ''' obtinentur sequentes tres aequationes

$$\varsigma'=-\frac{(\theta'+\theta'')\theta'\theta''}{2R''^3T''\gamma}\left(\sin\beta''\cos\beta''\sin(\alpha'-A'')-\sin\beta''\cos\beta'''\sin(\alpha'''-A'')\right)$$

$$-\frac{8R''}{3n^2T''\gamma}\left(\frac{\theta'^3}{(r'+r'')^3}-\frac{(\theta'+\theta'')^3}{(r'+r''')^3}+\frac{\theta''^3}{(r''+r''')^3}\right)\left(\sin\beta'''\cos\beta''\sin(\alpha''-A'')-\sin\beta''\cos\beta'''\sin(\alpha''-A'')\right)$$

$$-\frac{8\left(\frac{\theta''^2}{(r''+r''')^3}-\frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''}{3R''n^2T''\gamma}\left(\sin\beta'''\cos\beta''\cos(\alpha''-A'')-\sin\beta''\cos\beta'''\cos\alpha''-A''\right)$$

$$\varsigma''=\left[\frac{(\theta'+\theta'')\theta'\theta''}{2R''^3T'''\gamma}+\frac{8R''}{3n^2T'''\gamma}\left(\frac{\theta'^3}{(r'+r'')^3}-\frac{(\theta'+\theta'')^3}{(r'+r''')^3}+\frac{\theta''^3}{(r''+r''')^3}\right)\right]\left[\sin\beta'\cos\beta'''\sin(\alpha'''-A'')\ \cancel{\cos\beta'''}\right.$$
$$\left.-\sin\beta'''\cos\beta'\sin(\alpha'-A'')\right]$$

$$+\frac{8}{3R''n^2T'''\gamma}\left(\frac{\theta''^2}{(r''+r''')^3}-\frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''\left(\sin\beta'\cos\beta'''\cos(\alpha'''-A'')-\sin\beta'''\cos\beta'\cos\alpha'-A''\right)$$

$$\varsigma'''=-\left[\frac{(\theta'+\theta'')\theta'\theta''}{2R''^3T'\gamma}+\frac{8R''}{3n^2T'\gamma}\left(\frac{\theta'^3}{(r'+r'')^3}-\frac{(\theta'+\theta'')^3}{(r'+r''')^3}+\frac{\theta''^3}{(r''+r''')^3}\right)\right]\left[\sin\beta''\cos\beta'\sin(\alpha'-A'')\right.$$
$$\left.-\sin\beta'\cos\beta''\sin(\alpha''-A'')\right]$$

$$+\frac{8}{3R''n^2T'\gamma}\left(\frac{\theta''^2}{(r''+r''')^3}-\frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''\left(\sin\beta''\cos\beta'\cos(\alpha'-A'')-\sin\beta'\cos\beta''\cos(\alpha''-A'')\right)$$

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Seu posito

$$\mu' = \sin\beta'''\cos\beta''\sin(\alpha''-A'') - \sin\beta''\cos\beta'''\sin(\alpha'''-A'')$$

$$\mu'' = \sin\beta'\cos\beta'''\sin(\alpha'''-A'') - \sin\beta'''\cos\beta'\sin(\alpha'-A'')$$

$$\mu''' = \sin\beta''\cos\beta'\sin(\alpha'-A'') - \sin\beta'\cos\beta''\sin(\alpha''-A'')$$

$$\nu' = \sin\beta'''\cos\beta''\cos(\alpha''-A'') - \sin\beta''\cos\beta'''\cos(\alpha'''-A'')$$

$$\nu'' = \sin\beta'\cos\beta'''\cos(\alpha'''-A'') - \sin\beta'''\cos\beta'\cos(\alpha'-A'')$$

$$\nu''' = \sin\beta''\cos\beta'\cos(\alpha'-A'') - \sin\beta'\cos\beta''\cos(\alpha''-A'')$$

Sequentes tres

$$\rho' = -\left(\frac{(\theta'+\theta'')\theta'\theta''}{2R''^3 T''\gamma} + \frac{8R''}{3n^2 T''\gamma}\left(\frac{\theta'^3}{(r'+r'')^3} - \frac{(\theta'+\theta'')^3}{(r'+r''')^3} + \frac{\theta''^3}{(r''+r''')^3}\right)\mu'\right.$$

$$\left. + \frac{8}{3R''n^2 T''\gamma}\left(\frac{\theta''^2}{(r''+r''')^3} - \frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''\nu'\right.$$

$$\rho'' = \left(\frac{(\theta'+\theta'')\theta'\theta''}{2R''^3 T'''\gamma} + \frac{8R''}{3n^2 T'''\gamma}\left(\frac{\theta'^3}{(r'+r'')^3} - \frac{(\theta'+\theta'')^3}{(r'+r''')^3} + \frac{\theta''^3}{(r''+r''')^3}\right)\mu''\right.$$

$$\left. + \frac{8}{3R''n^2 T'''\gamma}\left(\frac{\theta''^2}{(r''+r''')^3} - \frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''\nu''\right.$$

$$\rho''' = -\left(\frac{(\theta'+\theta'')\theta'\theta''}{2R''^3 T'\gamma} + \frac{8R''}{3n^2 T'\gamma}\left(\frac{\theta'^3}{(r'+r'')^3} - \frac{(\theta'+\theta'')^3}{(r'+r''')^3} + \frac{\theta''^3}{(r''+r''')^3}\right)\mu'''\right.$$

$$\left. - \frac{8}{3R''n^2 T'\gamma}\left(\frac{\theta''^2}{(r''+r''')^3} - \frac{\theta'^2}{(r'+r'')^3}\right)\theta'\theta''\nu'''\right.$$

Porro quantitates Δ', Δ'', Δ''' continent tantum tertiam et quartam dimensionem de θ, quamobrem sufficiet primos tantum terminos quantitatum T', T'', T''' accipere: ex reali enim quantitatis Δ divisione per T nanciscimur

quotum usque ad quartam dimensionem exactum, terminus autem secundus valoris T cum sit tertiae dimensionis minimus terminus quem producet in divisione necessario erit quintae dimensionis, unius ratio haberi nequit; hac animadversione facta erit necessario $T' = \theta'$; $T'' = \theta''$ et $T''' = \theta'+\theta''$, quibus valoribus in ρ', ρ'', ρ''' substitutis obtinebuntur valores earundem quantitatum ρ'; ρ'' et ρ''' usque ad quartam dimensionem exacti.

§ 31. Sit nunc $\theta'' = \lambda\theta'$, et radius vector r'' spectetur ut functio temporis θ a certo termino tanquam epoche computati; haec igitur cum ejusmodi esse debeat, ut abeunte θ in $\theta-\theta'$ abeat r'' in r' et abeunte θ in $\theta+\theta''$ abeat r'' in r''', erit iuxta formulam Taylorianam

$$r' = r'' - \theta'\frac{dr''}{d\theta} + \&c$$

$$r''' = r'' + \theta''\frac{dr''}{d\theta} + \&c$$

Hoc posito erit

$$\frac{\theta'^3}{(r'+r'')^3} - \frac{(\theta'+\theta'')^3}{(r'+r''')^3} + \frac{\theta''^3}{(r''+r''')^3} = \frac{\theta'^3}{(2r'' - \theta'\frac{dr''}{d\theta})^3} - \frac{\theta'^3(\lambda+1)^3}{(2r'' + \lambda\theta'\frac{dr''}{d\theta} - \theta'\frac{dr''}{d\theta})^3} + \frac{\lambda^3\theta'^3}{(2r'' + \lambda\theta'\frac{dr''}{d\theta})^3}$$

$$\frac{\theta''^2}{(r''+r''')^3} - \frac{\theta'^2}{(r'+r'')^3} = \frac{\lambda^2\theta'^2}{(2r'' + \lambda\theta'\frac{dr''}{d\theta})^3} - \frac{\theta'^2}{(2r'' - \theta'\frac{dr''}{d\theta})^3}$$

negligendo nimirum terminos altiorum potestatum prima; His valoribus in ρ', ρ'', ρ''' introductis eaedem hae quantitates terminorum $(r'+r'')^3$, &c ρ', ρ'', ρ''' quae pro completo valore sunt usque ad quartam dimensionem exactae non erunt iam nisi usque ad secundam dimensionem, ~~quantitatum exactae~~

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et si plane termini $\frac{\theta' dr''}{d\theta}$ rejiciantur, non nisi ad primam dimensionem quantitatis θ' exactae. Ex quo iam duo casus enascuntur:

1º Quando intervalla temporum θ' et θ'' sunt aequalia, aut valde parum inter se differentia; ~~et~~

2do Dum eadem haec intervalla sunt notabiliter ~~int~~ differentia.

In primo casu erit rejiciendo potestates prima ipsius θ' altiores

$$\frac{\theta'^3}{\left(2r'' - \frac{\theta' dr''}{d\theta}\right)^3} = \frac{\theta'^3}{8r''^3 - 12r''^2 \frac{\theta' dr''}{d\theta}}$$

$$\frac{\theta'^3}{\left(2r'' + \frac{\lambda\theta' dr''}{d\theta}\right)^3} = \frac{\theta'^3}{8r''^3 + 12r''^2 \frac{\theta' dr''}{d\theta}}$$

$$\frac{-\theta'^3(\lambda+1)^3}{\left(2r'' + \frac{\lambda\theta' dr''}{d\theta} - \frac{\theta' dr''}{d\theta}\right)^3} = -\frac{\theta'^3}{r''^3}$$

adeoque reducendo

$$\frac{\theta'^3}{\left(2r'' - \frac{\theta' dr''}{d\theta}\right)^3} - \frac{\theta'^3(\lambda+1)}{\left(2r'' + \frac{\lambda\theta' dr''}{d\theta} - \frac{\theta' dr''}{d\theta}\right)^3} + \frac{\theta'^3}{\left(2r'' + \frac{\lambda\theta' dr''}{d\theta'}\right)^3} = -\frac{3\theta'^3}{4r''^3}$$

$$\frac{\lambda^2\theta'^2}{\left(2r'' + \lambda\theta' \frac{dr''}{d\theta}\right)^3} \quad \frac{-\theta'^2}{\left(2r'' - \frac{\theta' dr''}{d\theta}\right)^3} = -\frac{3\theta'^3}{8r''^4} \cdot \frac{dr''}{d\theta}$$

et denique

$$\varrho' = -\frac{R''\mu'\theta'^2}{\gamma}\left(\frac{1}{R''^4} - \frac{2}{n^2 r''^3}\right)$$

$$\varrho'' = \frac{R''\mu''\theta'^2}{2\gamma}\left(\frac{1}{R''^4} - \frac{2}{n^2 r''^3}\right)$$

$$\varrho''' = -\frac{R''\mu'''\theta'^2}{\gamma}\left(\frac{1}{R''^4} - \frac{2}{n^2 r''^3}\right)$$

neglectis nempe terminis in quibus θ' altioris est tertiae dimensionis; patet autem ex iis quae dicta sunt hos valores de $\varrho', \varrho'', \varrho'''$ esse exactos usque ad secundam dimensionem quantitatis θ'.

At pro secundo casu, in quo nempe non est $\lambda = 1$, abjectis terminis $\frac{\theta' dr''}{d\theta}$, habebitur

$$\frac{\theta'^3}{(r'+r'')^3} - \frac{(\theta'+\theta'')^3}{(r'+r''')^3} + \frac{\theta''^3}{(r''+r''')^3} = \frac{\theta'^3}{(2r'')^3} - \frac{\theta'^3(\lambda+1)^3}{(2r'')^3} + \frac{\lambda^3\theta'^3}{(2r'')^3}$$

$$\frac{\theta''^2}{(r''+r''')^3} - \frac{\theta'^2}{(r'+r'')^3} = \frac{\lambda^2\theta'^2}{(2r'')^3} - \frac{\theta'^2}{(2r'')^3}$$

et debitis substitutionibus institutis

$$\varrho' = -\frac{(1+\lambda)\,\cancel{\ }\,.R''\mu'\theta'^2}{2\gamma}\left(\frac{1}{R''^4} - \frac{2}{n^2 r''^3}\right)$$

$$\varrho'' = \frac{\lambda R''\mu''\theta'^2}{2\gamma}\left(\frac{1}{R''^4} - \frac{2}{n^2 r''^3}\right)$$

$$\varrho''' = -\frac{(1+\lambda)R''\mu'''\theta'^2}{2\gamma}\left(\frac{1}{R''^4} - \frac{2}{n^2 r''^3}\right)$$

qui valores non sunt nisi ad primam dimensionem quantitatis θ' exacti

32. Inventis hac ratione valoribus $\varrho', \varrho'', \varrho'''$ in quibus ~~rx~~ praeter r'' nulla incognita occurrit, poterit iam haec quoque ope aequationis $r''^2 = R''^2 + 2\varrho'' R'' \cos(\alpha'' - A'')\cos\beta'' + \varrho''^2$ eliminari, hac autem operatione instituta quae non differt ab illa Nri 27. pervenietur ad aequationem octavi gradus, sed quae posito $R'' = 1$ deprimetur ad gradum septimum

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quaeque adeo necessario habet unam radicem realem. Ceterum valores hi de $\varrho', \varrho'', \varrho'''$ sunt tantum ad primam vel secundam dimensionem quantitatis θ' exacti, prouti nimirum problema est casus primi vel secundi: illud autem per se manifestum est, si λ fuerit proxime unitati aequale, problema quidem pertinere ad casum secundum, sed valores ~~non~~ de $\varrho', \varrho'', \varrho'''$ non esse perfecte exactos ad dimensionem secundam ipsius θ' sed proxime tantum. Ad hunc igitur postremum casum pertinet determinatio orbitae cometae, ex observationibus ~~quas~~ Wargentin anno 1771 ~~observavit~~, factis. cum enim prima observatio facta sit ~~30~~ Aprilis 30^d 9^h, $48'$ $42''$.

Secunda ... Maji 8^d 10^h $24'$ $48''$

Tertia -- Maji 16^d 10^h $54'$ $45''$

pro Meridiano Stokholmiae, est in hoc casu

$\theta' = 8^d\ 0^h\ 76'\ 6''$

$\theta'' = 8^d\ 0^h\ 70\ 7''$

ita ut si ponatur $\theta' = \theta''$ non sit error nisi ~~?~~ $\frac{5}{10000}$ unius secundi

33. Interea hoc nihil officit methodo, quod in tam difficili quaestione non obtineantur ~~illico~~ valores exacti; inventis enim primis valoribus de $\varrho', \varrho'', \varrho'''$, poterunt ~~[illegible]~~ ~~[illegible]~~ per methodum approximationis inveniri exactiores, inventisque exactioribus adhuc exactiores idque tamdiu continuari quamdiu libuerit; id quod alioquin ex Theoria aequationum est notissimum.

Articulus VI.

34. His iam expeditis nihil est facilius quam reliqua elementa orbitae cometarum determinare. Nam ut a linea duo cometae loca iungente seu a valore k incipianus, habetur n. 19.

$$\tfrac{1}{2}\sqrt{[(r'+r'')^2-k^2]} = \frac{2\sigma\tau}{1-\tau}$$

seu ob $2\sigma = r'+r''$ et $\tau = 1-\frac{2\upsilon}{3}$

$$\tfrac{1}{2}\sqrt{[(r'+r'')^2-k^2]} = \left(\frac{r'+r''}{2}\right)(3-2\upsilon)$$

hinc $(r'+r'')^2-k^2 = (r'+r'')^2(3-2\upsilon)^2$ et

$$k = (r'+r'')\sqrt{1-(3-\upsilon)^2}$$

35. Et quia $p = \dfrac{k^2-(r'-r'')^2}{r'+r''-\sqrt{[(r'+r'')^2-k^2]}}$ n. 19

habetur parameter

$$p = \frac{(r'+r'')^2(3-\upsilon)^2}{r'+r''-\cancel{?}(r'+r'')(3-\upsilon)} = \frac{(r'+r'')(3-\upsilon)^2}{\upsilon-2}$$

36. Adeoque ob $\cos\omega = \dfrac{\sqrt{[(r'+r'')^2-k^2]}}{2\sqrt{r'r''}}$ n. 19, erit

$$\cos\omega = \frac{(r'+r'')(3-\upsilon)}{2\sqrt{r'r''}}$$

37. Porro habetur n. 17. $\operatorname{Tang}\dfrac{\varphi'}{2} = \dfrac{\sqrt{r''}.\cos\omega - \sqrt{r'}}{\sqrt{r''}.\sin\omega}$

cum igitur $\cos\omega = \dfrac{(r'+r'')(3-\upsilon)}{2\sqrt{r'r''}}$ erit $\sin\omega = \sqrt{\left[1-\dfrac{(r'+r'')^2(3-\upsilon)^2}{4r'r''}\right]}$

adeoque

$$\operatorname{Tang}\frac{\varphi'}{2} = \frac{(r'+r'')(3-\upsilon)-2r'}{\sqrt{[4r'r''-(r'+r'')^2(3-\upsilon)^2]}}$$

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et

$$\varphi = 2\,\text{Arc tang}\,\frac{[(x'+z'')(3-v)-2x']}{\sqrt{(4x'z''-(x'+z'')^2(3-v)^2)}}$$

habetur igitur angulus quem radius vector x' facit cum radio perihelii; seu positio perihelii

38. Ad inveniendam inclinationem orbitae Cometae, observo, ex aequationibus (A) n. 12. cognitis q innotescere x, y, z; deinde vero quia habetur n. 22 $x = at + bu$, $y = ct + eu$, $z = ft + gu$, eliminatis t et u obtinebitur

$$z = \frac{(ag-bf)y - (cg-ef)x}{ae-be}$$

Sed n. 13. est

$$z = y \cos h \tang i - x \sin h \tang i$$

quare

$$\cos h \tang i = \frac{ag-bf}{ae-bc}$$

$$\tang i \sin h = \frac{cg-ef}{ae-bc}$$

i est inclinatio orbitae, h vero longitudo nodi ascendentis

Porro sumtis duabus observationibus habetur

$x' = at' + bu'$, $x'' = at'' + bu''$

$y' = ct' + eu'$, $y'' = ct'' + eu''$

$z' = ft' + gu'$, $z'' = ft'' + gu''$

unde

$$x''z' - x'z'' = (ag-bf)(t''u' - t'u'')$$

$$y''z' - y'z'' = (cg-ef)(t''u' - t'u'')$$

$$x''y' - x'y'' = (ae-bc)(t''u' - t'u'')$$

et facta substitutione

$$\cos h \tang i = \frac{x''z' - x'z''}{x''y' - x'y''}$$

$$\tang i \sin h = \frac{y''z' - y'z''}{x''y - x'y''}$$

Unde ob cognitas x'', x'; y'', y'; z'', z' innotescent etiam anguli i et h: posito enim brevitatis gratia $\cos h \tang i = M$ et $\tang i \sin h = N$ erit $\frac{N}{\sin h} = \frac{M}{\cos h}$, et $\frac{\sin h}{\cos h} = \frac{N}{M}$ adeoque $\tang h = \frac{N}{M}$. Cognita $\tang h$ datur $\cos h$, ac proinde etiam $\tang i$.

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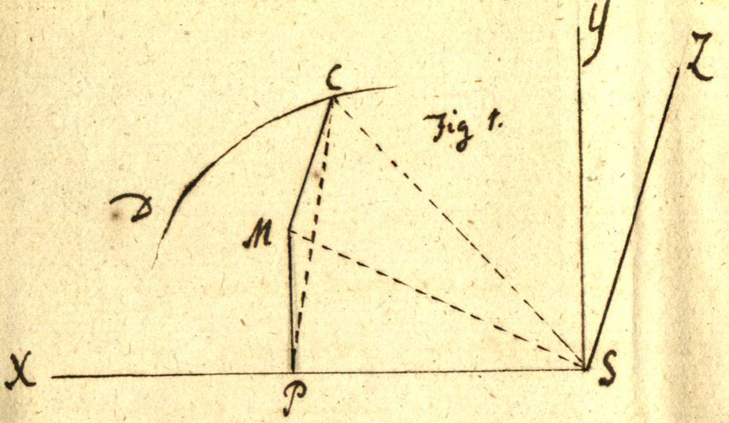

Fig 1.

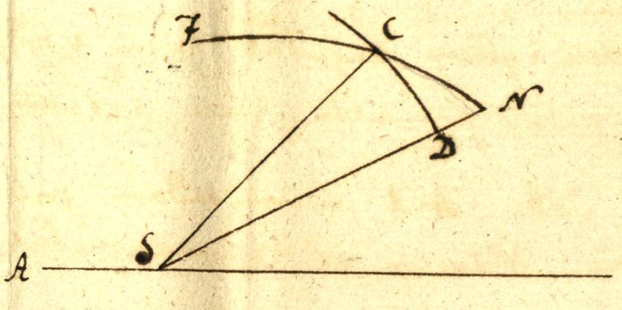

Fig 2.

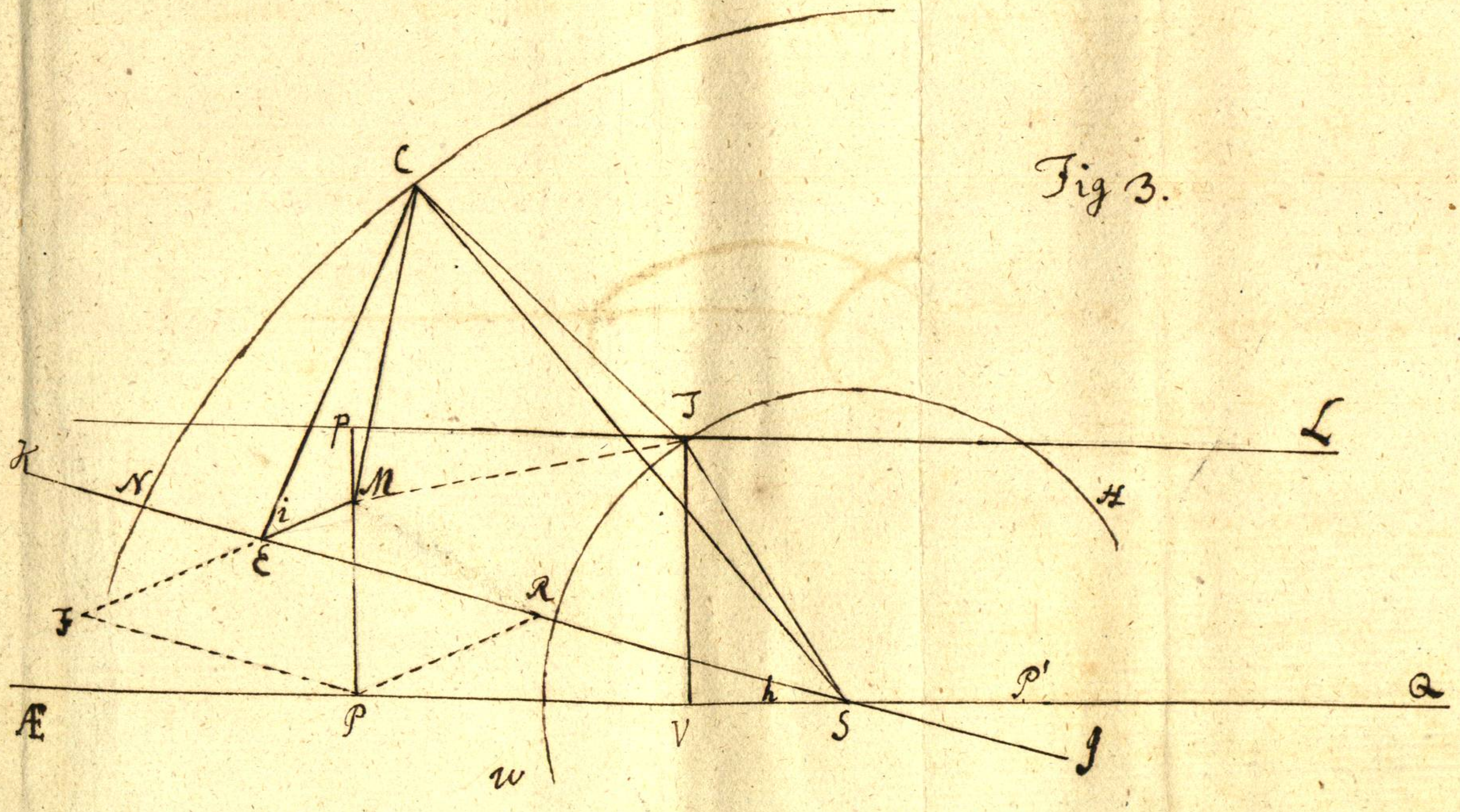

Fig 3.

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MNL OL, Magyar Királyi Helytartótanács, Departamentum litterario-politicum (C 67), 1796. F. 9, p. 218.

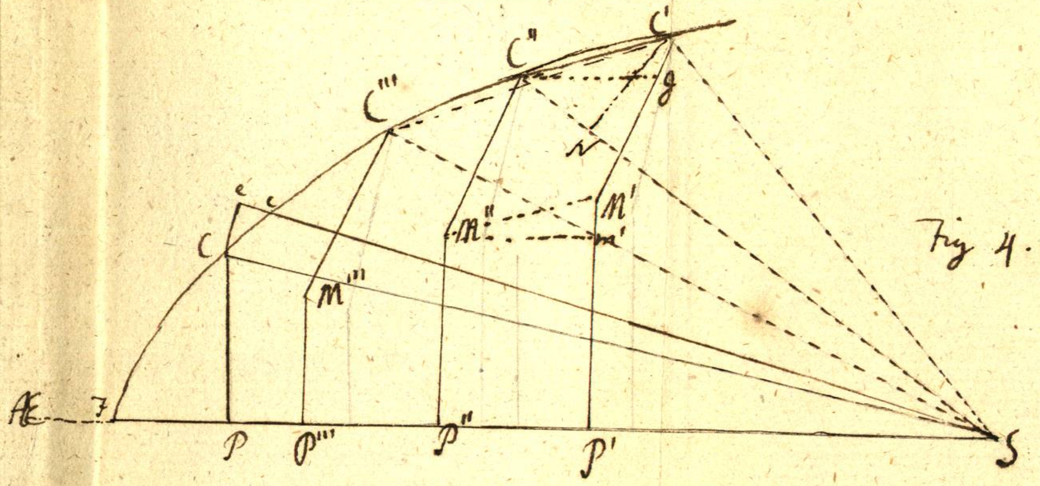

Fig 4.

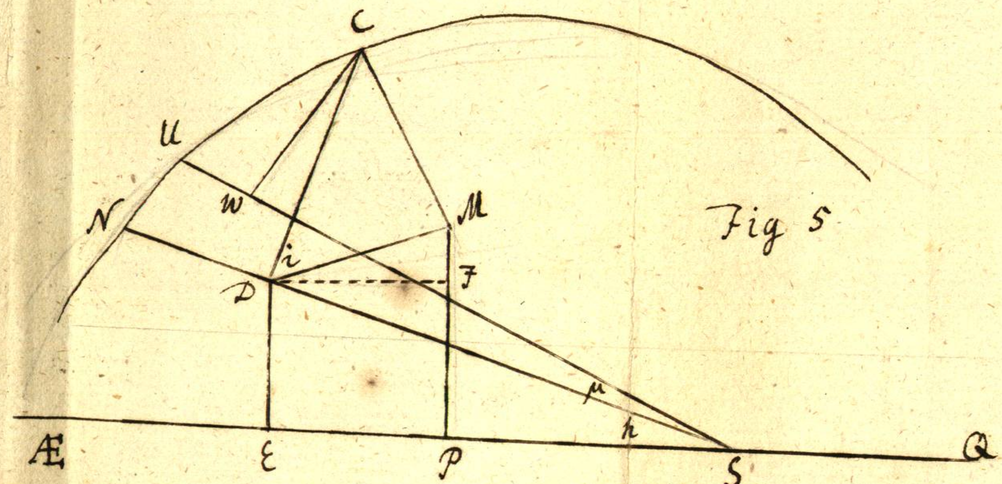

Fig 5

MNL OL, Magyar Királyi Helytartótanács, Departamentum litterario-politicum (C 67), 1796. F. 9, p. 218.